3. 

\[ L = 20 \text{ nH} \quad C = 200 \text{ pF} \]

\[ \begin{array}{c}
+ & \quad \text{R = 7.5 } \Omega & \quad + \\
\text{v}_i & \quad \quad & \quad \text{v}_o \\
- & \quad \quad & \quad -
\end{array} \]

For the band-pass filter shown above, calculate the following quantities:

a) \( \omega_0 \)

b) \( f_0 \)

c) \( \omega_{c1} \) and \( \omega_{c2} \)

d) \( \beta \) and \( Q \)

sol'n: a) For this standard form of filter, the center frequency is \( \omega_0 = \sqrt{\frac{1}{LC}} \).

\[
\omega_0^2 = \frac{1}{LC} = \frac{1}{20 \cdot 200} = \left(\frac{1}{20}\right)^2
\]

Note: \( \omega_0 \) is where \(|H(s)|\) = min or max. We accept that min or max of \(|H(s)|\) occurs where \( \omega = \omega_0 \). We could prove this by finding \( H(s) \).

\[
\omega_0 = \frac{1}{2n} \approx \frac{1}{2} \text{ G r/s or 500 M r/s}
\]
b) \( f_0 = \frac{\omega_0}{2\pi} \) since \( \omega = 2\pi f \) in general

\[ f_0 = \frac{500 \text{ MHz}}{2\pi} = 79.6 \text{ kHz} \]

c) We could use standard formulas for \( \omega_{c1} \) and \( \omega_{c2} \), but the derivation is instructive.

\( \omega_{c1,2} \) occur where \( |H(s)| = \frac{1}{\sqrt{2}} \max |H(s)| \).

\( H(s) \equiv \frac{V_o(s)}{V_c(s)} \) for frequency domain model:

\[ H(s) = \frac{R}{R + sL + \frac{1}{sC}} = \frac{1}{1 + sL + \frac{1}{sC}} \]

\[ H(j\omega) = \frac{1}{1 + j\omega L + \frac{1}{j\omega C}} = \frac{1}{1 + j(\omega L - \frac{1}{\omega C})} \]

At \( \omega_0 \) we have \( \omega L - \frac{1}{\omega C} = 0 \) and \( |H| = 1 \).

This is the maximum value \( |H| \) can have; this is the only frequency where the imaginary part of \( H \) is zero.
It follows that \( \omega_{c1/2} \) solve \( |R| = \frac{1}{\sqrt{2}} \cdot 1 = \frac{1}{\sqrt{2}} \).

\[
|H(j\omega_c)| = \frac{1}{\sqrt{2}} \left| 1 + j \left( \frac{\omega_c - \frac{1}{\omega_c}}{R} \right) \right| = \frac{1}{\sqrt{2}}.
\]

or \( \left| 1 + j \left( \frac{\omega_c - \frac{1}{\omega_c}}{R} \right) \right| = \sqrt{2} \).

The soln is \( \omega_c \frac{\omega_c - \frac{1}{\omega_c}}{R} = \pm 1 \)

or \( \omega_c \frac{\omega_c - \frac{1}{\omega_c}}{R} = \pm \omega_c \)

or \( \frac{\omega_c^2 - 1}{\omega_c} \frac{1}{R} = \pm \omega_c \)

or \( \omega_c^2 - \frac{1}{\omega_c} \frac{1}{R} = \pm \omega_c \frac{R}{\omega_c} \)

or \( \omega_c^2 - \frac{1}{\omega_c} \frac{R}{\omega_c} \frac{1}{L} = 0 \)

or \( \omega_c = \pm \frac{R}{2L} \pm \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}} \)

Since \( \omega_c \)'s > 0 and \( \sqrt{\text{term}} > \frac{R}{2L} \), we use

\[
\omega_{c1/2} = \pm \frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}
\]

Now for the numbers:

\[
\frac{R}{2L} = \frac{7.5 \text{ r} \cdot \text{r}}{2 \cdot 20 \text{ r} \cdot \text{r}} = \frac{7.5 \cdot 25}{40 \cdot 25} = 187.5 \text{ M r/s}
\]
\[
\sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = \sqrt{(187.5 \text{M})^2 + (500 \text{M})^2} \text{ r/s}
\]

= 534 \text{ M r/s}

Thus \( \omega_{c1,2} = \pm 187.5 \text{ M} + 534 \text{ M} \text{ r/s} \)

\( \omega_{c1} = 346.5 \text{ Mr/s} \)

\( \omega_{c2} = 721.5 \text{ Mr/s} \)

d) Bandwidth \( \beta = \omega_{c2} - \omega_{c1} = \frac{R}{L} = \frac{7.5}{20n} \text{ r/s} \)

\( \beta = \frac{7.5 \cdot 50}{20n \cdot 50} = 375 \text{ M r/s} \)

Quality factor \( Q = \frac{\omega_0}{\beta} = \frac{500 \text{ Mr/s}}{375 \text{ Mr/s}} = \frac{\pi}{2} \)