IDENTITY: \[ \mathcal{L}\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right) \text{ for } a > 0 \]

PROOF: By definition, we have the following equation:
\[ \mathcal{L}\{f(at)\} = \int_{0}^{\infty} f(at)e^{-st} \, dt \]

We change variables to \( \tau = at \).

At \( t = 0^- \), \( \tau = a0^- = 0^- \).

For \( t \to \infty \), \( \tau = a\infty = \infty \).

Inside the integral, \( st = s\tau/a \).

For \( dt \) we have \( dt/d\tau = a \), so \( dt = d\tau/a \).

Making these substitutions, our identity is verified:
\[ \mathcal{L}\{f(at)\} = \int_{0}^{\infty} f(\tau)e^{-s\tau/a} \frac{d\tau}{a} = \frac{1}{a} F\left(\frac{s}{a}\right). \]

NOTE: \( \tau \) is a variable of integration that we may replace with \( t \) in the final integral.