IDENTITY: \[ \mathcal{L}^{-1}\left\{ \frac{1}{(s + a)^n} \right\} = \frac{t^{n-1}}{(n-1)!} e^{-at} \]

PROOF: Use induction.

First, verify the identity for \( n = 1 \). From straightforward calculations, the following result is known:

\[ \mathcal{L}\{e^{-at}\} = \frac{1}{s + a} \]

For \( n = 1 \), the identity gives the same result:

\[ \mathcal{L}^{-1}\left\{ \frac{1}{(s + a)^1} \right\} = \frac{t^{1-1}}{(1-1)!} e^{-at} = e^{-at} \]

NOTE: \( 0! = 1 \).

Thus, the identity is valid for \( n = 1 \).

Now assume the identity is valid for \( n > 1 \) and show that it holds for \( n + 1 \).

Thus, we assume the following is true:

\[ \mathcal{L}^{-1}\left\{ \frac{1}{(s + a)^n} \right\} = \frac{t^{n-1}}{(n-1)!} e^{-at} \]

or

\[ \mathcal{L}\left\{ \frac{t^{n-1}}{(n-1)!} e^{-at} \right\} = \frac{1}{(s + a)^n} \]

Apply the following identity for Laplace transforms:

\[ \mathcal{L}\{tf(t)\} = -\frac{dF(s)}{ds} \quad \text{where} \quad F(s) = \mathcal{L}\{f(t)\} \]

This yields the following result:

\[ \mathcal{L}\left\{ t \frac{t^{n-1}}{(n-1)!} e^{-at} \right\} = -\frac{d}{ds} \frac{1}{(s + a)^n} = -\frac{-n}{(s + a)^{n+1}} = \frac{n}{(s + a)^{n+1}} \]

Since the Laplace transform is linear, we have the following identity:
\[ \mathcal{L} \left\{ \frac{1}{n} g(t) \right\} = \frac{1}{n} G(s) \]

Applying this to our last result yields an equation that matches the identity we are trying to prove when \( n + 1 \) is substituted for \( n \):

\[ \mathcal{L} \left\{ \frac{t^{n-1} e^{-at}}{n (n-1)!} \right\} = \frac{1}{(s + a)^{n+1}} \]

or

\[ \mathcal{L} \left\{ \frac{t^{(n+1)-1}}{((n+1)-1)!} e^{-at} \right\} = \frac{1}{(s + a)^{n+1}} \]

By the axiom of induction, it follows that the identity holds for all \( n \geq 1 \), and our proof is complete.