Ex:

Ex:  

\[ L = 4 \, \mu H \]

\[ 4 \, \mu F \]

\[ R = 0.625 \, \Omega \]

a) Find the characteristic roots, \( s_1 \) and \( s_2 \), for the above circuit.

b) If \( C \) is reduced to 1 nF, which will the circuit be: over-damped or under-damped?

c) Using the \( L \) and \( C \) values in the circuit, what value of \( R \) yields a critically-damped circuit?

**SOL'N:** a) For a parallel RLC circuit (or a series RLC circuit), we have the following formula for the characteristic roots:

\[ s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} \]

For a parallel RLC circuit, the value of \( \alpha \) is one-half the inverse RC time constant:

\[ \alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 0.625 \Omega \cdot 4 \, \mu F} = \frac{1 \, M/s}{5} = 200 \, k/s \]

For both parallel and series RLC circuits, the resonant frequency, \( \omega_o \), is the inverse of the square root of the product of \( L \) and \( C \):

\[ \omega_o = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \omega_o^2 = \frac{1}{LC} = \frac{1}{4 \, \mu H \cdot 4 \, \mu F} = \left(\frac{1}{4 \, \mu \text{ r/s}}\right)^2 = (250 \, \text{kr/s})^2 \]

We find that, since \( \alpha < \omega_o \), the roots are complex:

\[ s_{1,2} = -200 \, \text{kr/s} \pm \sqrt{(200 \, \text{kr/s})^2 - (250 \, \text{kr/s})^2} = -200 \, \text{kr/s} \pm j150 \, \text{kr/s} \]

When the roots are complex, the circuit is under-damped.
**SOL’N:** b) For \( C = 1 \text{ nF} \), we repeat the above calculations:

\[
\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 0.625 \Omega \cdot \text{lnF}} = \frac{\text{lk M/s}}{1.25} = 800 \text{ M/s}
\]

\[
\omega_o = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \omega_o^2 = \frac{1}{LC} = \frac{1}{4 \text{ \mu H} \cdot \text{1 nF}} = 250 \text{ M}^2 (\text{r/s})^2
\]

or

\[
s_{1,2} = -800 \text{ Mr/s} \pm \sqrt{800^2 \text{ M}^2 (\text{r/s})^2 - 250 \text{ M}^2 (\text{r/s})^2}
\]

or

\[
s_{1,2} = -800 \text{ Mr/s} \pm 799.8 \text{ Mr/s}
\]

or

\[
s_1 = -0.2 \text{ M/s} \quad \text{and} \quad s_2 = -1.599 \text{ G/s}
\]

When the roots are real, the circuit is over-damped.

**NOTE:** The second root corresponds to a decay that is extremely rapid compared to that for the first root. Consequently, the response of the circuit after the first few nanoseconds looks like an RC exponential decay with time constant equal to \(-1/s_1\). This means the inductor is so small it has little effect on the response of the remaining RC circuit.

**SOL’N:** c) For critical damping, we will have \( \alpha = \omega_o \). From part (a), we have the value of \( \omega_o \), (which is independent of \( R \)):

\[
\omega_o = 250k \text{ r/s}
\]

Using this value, we solve for \( R \) via \( \alpha \):

\[
\alpha = \frac{1}{2RC} = \omega_o = 250k \text{ r/s}
\]

or

\[
R = \frac{1}{2\omega_o C} = \frac{1}{2 \cdot 250k \cdot 4\mu} \Omega = \frac{1}{2000k \mu} \Omega = 0.5 \Omega
\]