Ex:

After being open for a long time, the switch closes at $t = 0$.

Give expressions for the following in terms of at most $i_g$, $R_1$, $R_2$, $L$, and $C$:

$$i(t = 0^+) \quad \text{and} \quad \frac{di(t)}{dt} \bigg|_{t=0^+}$$

SOL'N:

Start with the circuit at $t=0^-$ and find energy variables, $i_L$ and $V_C$, that stay the same when the switch closes. Treat $L$ as a wire and $C$ as an open circuit.

$t=0^-$:

- $i_L(0^-) = 0A$
  - since $C$ = open
- $V_C(0^-) = i_g R_2$ since all $i_g$ goes thru $R_2$
At $t=0^+$, treat $L$ as current source and $C$ as voltage source. Use same values as at $t=0^-$. Switch closed.

$t=0^+$:

Since $L$ acts like an open circuit,

$i(0^+) = i_L(0^+) = 0 \text{ A}$

To find $\frac{di}{dt}$, we first write $i(t)$ in terms of energy (or state) vars $v_C$ and $i_L$. (Note: our eqn must apply for any $t > 0$, so we do not plug in values for $v_C$ and $i_L$. These values remain symbolic.) We draw the circuit as follows:

Here, it is easy to write $i$ in terms of $i_L$ and $v_C$.

$i = i_L$
Now we differentiate our eq'n for $i$:

$$\frac{di}{dt} = \frac{di}{dt} = \frac{v_L}{L}$$

Finally, we have an expression without a derivative in it, and we may plug in values:

$$\frac{di}{dt} \bigg|_{t=0^+} = \frac{v_L}{L} \bigg|_{t=0^+}$$

From a voltage loop eq'n, we have the following result:

$$v_L = -\left[i_L(0^+)R_1 + v_c(0^+)\right]$$

$$v_L = -\left[0 \cdot R_1 + i_g R_2\right] = -i_g R_2$$

$$\therefore \frac{di}{dt} \bigg|_{t=0^+} = -\frac{i_g R_2}{L}$$