Ex:

At \( t = 0 \), \( v_g(t) \) switches instantly from \(-v_o\) to \( v_o\).

a) Write the state-variable equations for the circuit in terms of the state vector:

\[
\dot{x} = \begin{bmatrix} i_1 \\ v_1 \\ v_2 \end{bmatrix}
\]

b) Evaluate the state vector at \( t = 0^+\).

**Sol'n:**

a) We have first derivatives of state vars on the left, and we first equate these to non-derivatives:

\[
\frac{di_1}{dt} = \frac{di_1}{dt} = \frac{v_i}{L}
\]

\[
\frac{dv_1}{dt} = \frac{dv_{C_1}}{dt} = \frac{i_{C_1}}{C_1}
\]

\[
\frac{dv_2}{dt} = \frac{dv_{C_2}}{dt} = \frac{i_{C_2}}{C_2}
\]

We now write the variables on the right in terms of only state vars \( i_1, v_2 \), and \( v_3 \), (and components and sources).
This problem of writing $i'$s and $v'$s in terms of state vars is easier to solve if we redraw the circuit with $i_L$'s and $v_o$'s shown as sources.

Note: we use $v_o = +v_o$ for $t > 0$.

Using superposition, we turn on one source at a time.

I. $i_1$ on

$v_{L1} = 0V \quad (c_1, c_2 \text{ form wire, short})$

$i_{c1} = -i_1 \quad (\text{all of } i_1 \text{ flows in short})$

$i_{c2} = -i_1 \quad (\text{all of } i_1 \text{ flows in short})$
II. \( v_g = v_o \) on

\[ V_{L2} = 0 \text{V} \quad \text{(shorted by } C_1, C_2) \]

\[ i_{c12} = \frac{v_o}{R_1} \quad \text{(middle loop carries all of the current)} \]

\[ i_{c22} = \frac{v_o}{R_1} \quad \text{(ditto, } R_2 \text{ shorted)} \]

III. \( v_1 \) on

\[ V_{L3} = v_1 \quad \text{(} L \text{ is connected across } v_1 \text{)} \]

\[ i_{c13} = -\frac{v_1}{R_1} \quad \text{(all current is in middle loop)} \]

\[ i_{c23} = i_{c13} = -\frac{v_1}{R_1} \]
IV. $V_2$ on

$$V_{L4} = V_2 \quad (L \text{ is across } V_2)$$

$$i_{c14} = -\frac{V_2}{R_1} \quad (i \text{ flows around middle loop})$$

$$i_{c24} = -\frac{V_2}{R_1 + R_2} \quad (i \text{ in } R_1 \& R_2 \text{ from Ohm's law})$$

Combining results:

$$\frac{di_1}{dt} = \frac{V_1 + V_2}{L}$$

$$\frac{dv_1}{dt} = \frac{-i_1 - V_1 - V_2 + V_0}{R_1 + \frac{R_1}{R_2}}$$

$$\frac{dv_2}{dt} = \frac{-i_1 - V_1 - V_2 - V_2 + V_0}{R_1 + \frac{R_1}{R_2}}$$

b)

For initial conditions, we use the circuit at $t=0^-$, since $i_L(0^+) = i_L(0^-)$ and $V_c(0^+) = V_c(0^-)$. At $t=0^-$, $L = \text{wire}$, $C = \text{open}$, and $V_g = -V_0$. 
From loop on left, \( i_1(0^-) = -\frac{v_0}{R_1} \).

Note: no current flows on right side.

No current \( R_2 \), so \( v_2(0^-) = 0V \) is \( V \)-drop across \( R_2 \).

Short on left side and no \( V \)-drop for \( R_2 \) give \( v_1 = 0V \) from outer \( V \)-loop.

\[
\begin{align*}
  i_1(0^+) &= -\frac{v_0}{R_1} \quad \text{(same values as at } t=0^-) \\
  v_1(0^+) &= 0V \\
  v_2(0^+) &= 0V
\end{align*}
\]