Ex:

One period, $T$, of a function $v(t)$ is shown above. The formula for $v(t)$ is

$$v(t) = \begin{cases} 
-4 \text{ V} & 0 < t < T/4 \\
4 \text{ V} & T/4 < t < T/2 \\
-4 \text{ V} & T/2 < t < 3T/4 \\
-12 \text{ V} & 3T/4 < t < T 
\end{cases}$$

Find the numerical value of the following coefficients of the Fourier series for $v(t)$: (Hint: remove the DC offset after finding its value.)

a) $a_N$  
b) $a_1$

Ex: Find the value of $b_1$ and $b_2$ for the Fourier series.
sol'n: a) By inspection, the average value of the waveform is $-4V$. One way to see this is to imagine moving the block from $T/4$ to $T/2$ above $-4V$ to $3T/4$ to $T$:

![Waveform Diagram]

If we wish to use an integral, we have the following:

\[
\overline{V} = \frac{1}{T} \int_0^T v(t) \, dt
\]

\[
= \frac{1}{T} \left( \int_0^{T/4} -4V \, dt + \int_{T/4}^{T/2} 4V \, dt + \int_{T/2}^{3T/4} -4V \, dt + \int_{3T/4}^T -12V \, dt \right)
\]

\[
= \frac{1}{T} \left( -4V \frac{T}{4} + 4V \frac{T}{2} -4V \frac{3T}{4} -12V \frac{T}{4} \right)
\]

\[
= \frac{1}{T} \left( -4V \frac{T}{4} + 4V \frac{T}{2} -4V \frac{T}{4} -12V \frac{T}{4} \right)
\]

\[
= -16V
\]

\[
\overline{V} = -4V
\]
Another approach would be to remove the DC offset from $v(t)$ before calculating $a_1$.

\[ v(t) - a_V \]

We double the area from $\frac{T}{4}$ to $\frac{T}{2}$:

\[
a_1 = 2 \cdot \frac{2}{T} \int_{T/4}^{T/2} 8V \cos\left(\frac{2\pi t}{T}\right) dt
\]

\[
= \frac{4}{T} \cdot 8V \left[ \sin\left(\frac{2\pi t}{T}\right) \right]_{T/4}^{T/2}
\]

\[
= \frac{16V}{\pi} \left[ \sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{4}\right) \right]
\]

\[
a_1 = -\frac{16V}{\pi}
\]
\[ b_1 = \frac{2}{T} \int_0^T v(t) \sin(w_0 t) \, dt \]

We sketch \([v(t) - a_V] \sin(w_0 t)\). That is, we remove the DC offset of \(v(t)\).

We double the area from \(\frac{T}{4}\) to \(\frac{T}{2}\):

\[ b_1 = 2 \cdot \frac{2}{T} \int_{T/4}^{T/2} 8V \sin\left(\frac{2\pi t}{T}\right) \, dt \]

\[ = \frac{32V}{T} \left[ -\cos\left(\frac{2\pi t}{T}\right) \right]_{T/4}^{T/2} \]

\[ = \frac{16V}{T} (-1 - 0) \]

\[ = \frac{16V}{T} \]

We could also have observed that the areas are just negatives of the areas sketched for the calculation of \(b_1\). Thus, \(b_1 = -a_1 = \frac{16V}{\pi}\).
\[ b_2 = \frac{2}{T} \int_{0}^{T} v(t) \sin(2\omega_0 t) \, dt \]

we sketch \([v(t) - a_v] \sin(2\omega_0 t)\).

\[ \begin{array}{c}
\text{The areas cancel. Thus,}
\end{array} \]

\[ b_2 = 0 \, \text{V} \]