Ex: Find the Laplace transform of the following waveform:

\[ f(t) = e^{-a(t-\tau)}u(t-\tau) \quad \text{where } \tau > 0 \]

Soln: Use the identity for signals with a delayed turn-on and which are shifted in time:

\[ \mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s), \quad a > 0 \]

where

\[ F(s) \equiv \mathcal{L}\{f(t)\} = \int_{0^-}^{\infty} f(t)e^{-st} dt \]

From the step function, \( u(t-\tau) \), we identify \( \tau \) as the value of \( a \). To use the identity, we must identify \( f(t-\tau) \). The exponential is already in the form of \( f(t-\tau) \):

\[ f(t-\tau) = e^{-a(t-\tau)} \]

For the delay identity, we need \( f(t) \). We obtain \( f(t) \) by replacing \( t-\tau \) with \( t \).

\[ f(t) = e^{-at} \]

We lookup the Laplace transform of \( e^{-at} \) in a table:

\[ \mathcal{L}\{e^{-at}\} = \frac{1}{s+a} \]

Our final result:

\[ \mathcal{L}\{e^{-a(t-\tau)}u(t-\tau)\} = e^{-\tau s} \cdot \frac{1}{s+a} \]