**Transform Pair:**
\[ \mathcal{L}\left\{ \frac{t^{n-1}}{(n-1)!} \right\} = \frac{1}{s^n} \]

**Proof:** We derive this transform pair by considering how we get \( \frac{1}{s^n} \) for values of \( n \) starting from \( n = 1 \).

\[ \mathcal{L}\{u(t)\} = \frac{1}{s} \]

We use the time-integral identity to obtain \( \frac{1}{s^2}, \frac{1}{s^3} \), and so forth.

\[ \mathcal{L}\left\{ \int_0^t f(\tau) d\tau \right\} = \frac{F(s)}{s} \]

**Note:** \( \tau \) is a variable of integration that plays the role of \( t \) to avoid confusion with the \( t \) in the upper limit of the integral.

Thus, we have

\[ \mathcal{L}\left\{ \int_0^t u(\tau) d\tau = t \right\} = \frac{1}{s} \cdot \frac{1}{s} = \frac{1}{s^2} \]

and

\[ \mathcal{L}\left\{ \int_0^t \tau d\tau = \frac{t^2}{2} \right\} = \frac{1}{s} \cdot \frac{1}{s^2} = \frac{1}{s^3} \]

and

\[ \mathcal{L}\left\{ \int_0^t \frac{\tau^2}{2} dt = \frac{t^3}{3!} \right\} = \frac{1}{s} \cdot \frac{1}{s^3} = \frac{1}{s^4} \]

If we continue, the pattern is the transform pair we seek. To prove the identity more formally, we use induction. The identity has been verified for the first few terms, and all that remains is to show that we can apply the time-integral identity to the transform pair for \( n \) to obtain the transform pair for \( n + 1 \).

\[ \mathcal{L}\left\{ \frac{t^{n-1}}{(n-1)!} \right\} = \frac{1}{s^n} \]
We apply the time-integral identity:

\[
\mathcal{L} \left\{ \int_0^\tau \frac{\tau^{n-1}}{(n-1)!} d\tau = \frac{\tau^n}{n!} \right\} = \frac{1}{s} \cdot \frac{1}{s^n}
\]

or

\[
\mathcal{L} \left\{ \frac{t^n}{n!} \right\} = \frac{1}{s^{n+1}}
\]

This equation matches the form of the transform pair, and the proof is complete.