Ex: \[ i_s(t) = \begin{cases} -1 \text{ A} & t < 0 \\ 1 \text{ A} & t \geq 0 \end{cases} \]

Find a symbolic expression for the Laplace-transformed output, \( V_o(s) \), in terms of not more than \( R_1, R_2, L, C \), and values of sources or constants.

a) Find a symbolic expression for the Laplace-transformed output, \( V_o(s) \), in terms of not more than \( R_1, R_2, L, C \), and values of sources or constants.

b) Choose a numerical value for \( R_1 \) to make

\[ v_1(t) = v_m e^{-\alpha t} \sin(\beta t) \]

where \( \beta = 7 \text{ k rad/s} \) and \( v_m \) and \( \alpha \) are real-valued constants.

Hint: \( R_1 \) behaves as though it is in parallel with \( L \) and \( C \).

c) Find the values of \( v_m \) and \( \alpha \).

soln: a) We find initial conditions at \( t=0^- \) for the \( L \) and \( C \) before transforming to the Laplace domain. Given that \( i_s(t) \) is constant at \(-1 \text{ A}\) for \( t<0 \), and \( R_1 \) behaves as though it is in parallel...
with \( L \) and \( C \) (as noted in the problem statement), we conclude that any oscillation will die out and leave the circuit in a constant state.

In other words, \( v_L = L \frac{di_L}{dt} = L \cdot 0 = 0 \)

and \( i_C = C \frac{dv_C}{dt} = C \cdot 0 = 0. \)

It follows that the \( L \) acts like a wire and the \( C \) acts like an open circuit.

In addition, the negative feedback of the op-amp causes the voltage at the input to be approximately \( 0V \), or a virtual reference.

\[ i_y(0^-) = -1A \]

The current follows the path of least resistance in choosing to flow thru the \( L \) rather than \( C \) or \( R \).
\[ i_L(0^-) = -1 \text{ A} \]

Since the \( L \) acts like a wire, the initial voltage on \( C \) is 0V:

\[ V_C(0^-) = 0 \text{ V} \]

Parallel sources for initial conditions are convenient in the \( \mathcal{F} \)-domain:

\[ \frac{1}{\mathcal{F}} \quad \text{and} \quad \frac{1}{\mathcal{F}} \]

We may omit the initial condition for \( C \), since it is zero.

For the Laplace transform of \( i_T(t) \), we use only the value of \( i_T(t) \) for \( t > 0 \). (The value of \( i_T(t) \) for \( t < 0 \) is accounted for by the initial conditions on \( L \) and \( C \).)

\[ I_T(s) = \mathcal{L}[1 \text{ A}] = \mathcal{L}[1 \text{ A} \cdot u(t)] = \frac{1}{s} \frac{1}{s} \]

\( \mathcal{F} \)-domain model:
We have $I_f(s) = I_3$, since no current flows into the op-amp. Since we have 0V at the -input of the op-amp, we have

$$V_o(s) = -I_f(s) \cdot R_2$$

or

$$V_o(s) = -I_3(s) \cdot R_2.$$ 

To find $I_3$, we use the virtual ref at the -input of the op-amp and place $R_1$ in parallel with the $L$ and $C$. We also combine current sources:

\[
I_3(s) = \frac{2}{5} A \cdot \frac{1}{sC} || \frac{sL}{R_1} \cdot \frac{V_i(s)}{R_1} \]

\[
V_o(s) = -I_3(s) \cdot R_2 = -\frac{2}{5} A \left( \frac{1}{sC} || \frac{sL}{R_1} \right) \cdot \frac{R_2}{R_1} \]
b) From part (a), we have $V_1(s)$:

$$V_1(s) = \frac{2}{s} A \left( \frac{1}{sC + \frac{1}{sL} \parallel R_1} \right)$$

$$V_1(s) = \frac{2}{s} A \cdot \frac{1}{sC + \frac{1}{sL} \parallel R_1}$$

$$V_1(s) = \frac{2}{s} \frac{Z/C}{s^2 + C \cdot \frac{1}{sL} + \frac{1}{R_1C}}$$. 

$$V_1(s) = \frac{\left(\frac{Z}{C} \cdot \frac{1}{sL}\right) \cdot \beta}{(s + \alpha)^2 + \beta^2}$$

where $\alpha \equiv \frac{1}{2R_1C}$, $\beta \equiv \frac{1}{LC} - \left(\frac{1}{Z/C}\right)^2$

Taking the inverse Laplace transform yields the following form of $V_1(t)$:

$$V_1(t) = \frac{2}{\beta C} e^{-\alpha t} \sin(\beta t)$$

We now find the value of $R_1$ that makes $\beta = 7 \text{ k rad/s}$.

$$\beta = \sqrt{\frac{1}{LC} - \left(\frac{1}{Z/C}\right)^2} = 7 \text{ k rad/s}$$

or

$$\beta^2 = \frac{1}{LC} - \left(\frac{1}{Z/C}\right)^2 = 49 \text{ M (rad/s)^2}$$
c) From the expression for $v_1(t)$ in part (b) we find the values of $v_m$ and $\alpha$.

\[ v_m = \frac{2}{\beta C} = \frac{2}{\frac{12}{7} \frac{1}{6} \mu} = \frac{12}{7} \text{kV} = 1.71 \text{kV} \]

\[ \alpha = \frac{1}{2R_1C} = \frac{1}{2 \cdot 125 \cdot \frac{1}{6} \mu} \text{r/s} = \frac{3M}{24} \text{r/s} = 24 \text{k r/s} \]