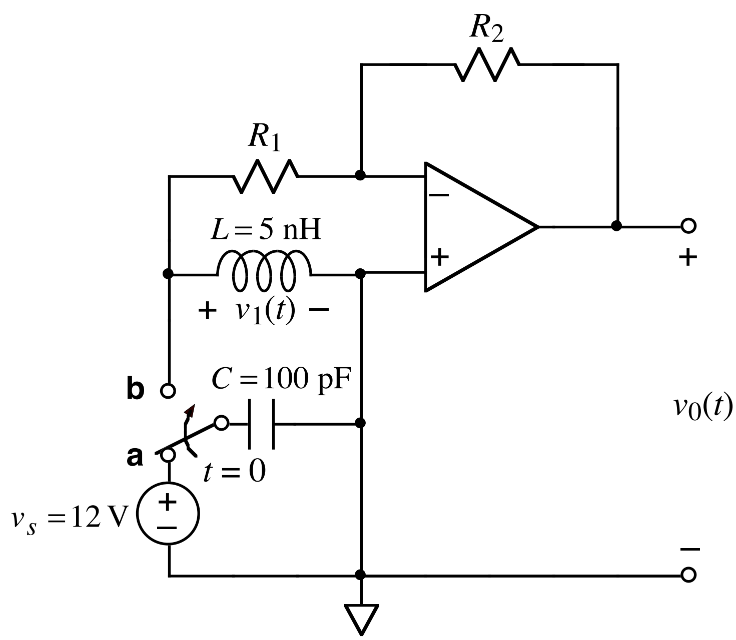
Ex:



After being in position **a** for a long time, the switch moves to position **b** at time $t = 0$.

a) Find a symbolic expression for the Laplace-transformed output, $\mathbf{V}_0(s)$, in terms of not more than R_1, R_2, L, C, and values of sources or constants.

b) Choose a numerical value for R_1 to make

$$v_1(t) = v_m e^{-\alpha t}[\cos(\beta t) - \sin(\beta t)]$$

where v_m, α, and β are real-valued constants.

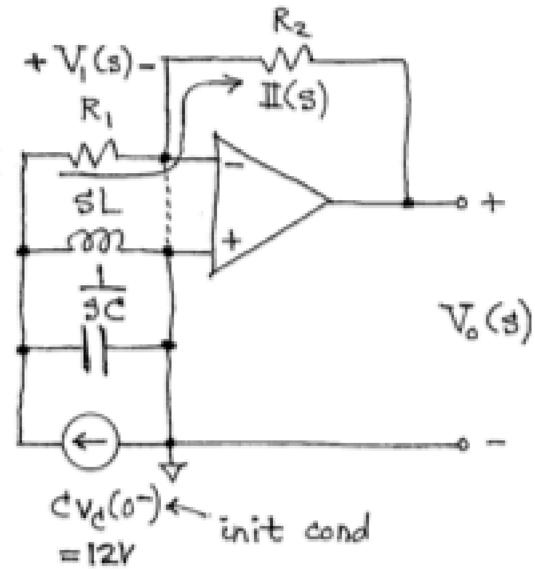
Hint: R_1 behaves as though it is in parallel with L and C.

Hint: $s = s + \alpha - \alpha$.

soln: a) First, we find the initial conditions. Clearly, $i_L(0^-) = 0A$ since there is no pwr source to drive L.

The C charges to 12V.

For the s-domain model, we may treat the − input as virtual ref. This means that R_1, L, C act as though they are in parallel.



$$V_0(s) = -I(s) R_2$$

We use an I-divider to find $I(s)$:

$$I(s) = C v_c(0^-) \cdot \frac{sL \parallel \frac{1}{sC}}{R_1 + sL \parallel \frac{1}{sC}}$$

where $sL \parallel \frac{1}{sC} = \frac{L/C}{sL + \frac{1}{sC}} = \frac{s/C}{s^2 + \frac{1}{LC}}$

We continue with the derivation by substituting the expression for $sL || \frac{1}{sC}$.

$$I(s) = C v_C(0^-) \frac{\dfrac{s/C}{s^2 + \frac{1}{LC}}}{R_1 + \dfrac{s/C}{s^2 + \frac{1}{LC}}}$$

or

$$I(s) = C v_C(0^-) \frac{s/C}{s^2 R_1 + \frac{1}{LC} R_1 + s/C}$$

or

$$I(s) = \cancel{C} v_C(0^-) \frac{s/R_1\cancel{C}}{s^2 + \frac{1}{R_1 C} s + \frac{1}{LC}}$$

Our final expression for $V_o(s)$:

$$V_o(s) = -v_C(0^-) \frac{R_2}{R_1} \frac{s}{s^2 + \frac{1}{R_1 C} s + \frac{1}{LC}}$$

b) We must match the form of $V_1(s)$ to the Laplace transform of $v_1(t)$:

$$V_1(s) = v_m \left[\frac{s+\alpha}{(s+\alpha)^2 + \beta^2} - \frac{\beta}{(s+\alpha)^2 + \beta^2} \right]$$

We have $V_1(s) = I(s) R_1$ for the circuit.

$$V_1(s) = v_C(0^-) \frac{s}{s^2 + \frac{1}{R_1 C} s + \frac{1}{LC}}$$

Equating denominators, we obtain expressions for α and β.

$$s^2 + \frac{1}{R_1 C} s + \frac{1}{LC} = (s+\alpha)^2 + \beta^2$$

$$= s^2 + 2\alpha s + \alpha^2 + \beta^2$$

Equating coefficients:

$$\alpha = \frac{1}{2R_1 C}, \quad \beta^2 = \frac{1}{LC} - \alpha^2$$

$$\text{or } \beta = \sqrt{\frac{1}{LC} - \alpha^2}$$

If we consider the numerator of $V_1(s)$, we need $s+\alpha-\beta$ but we have only s.

Using the hint that $s = s+\alpha-\alpha$, we can get $s+\alpha-\beta$ if

$$\alpha = \beta.$$

or

$$\frac{1}{2R_1 C} = \sqrt{\frac{1}{LC} - \left(\frac{1}{2R_1 C}\right)^2}$$

or

$$\left(\frac{1}{2R_1 C}\right)^2 = \frac{1}{LC} - \left(\frac{1}{2R_1 C}\right)^2$$

or

$$2\left(\frac{1}{2R_1 C}\right)^2 = \frac{1}{LC}$$

or

$$2R_1^2 C^2 = LC$$

or

$$R_1 = \sqrt{\frac{L}{2C}} = \sqrt{\frac{5\,\text{nH}}{2(100\,\text{pF})}} = \sqrt{25}\,\Omega$$

or

$$R_1 = 5\,\Omega$$