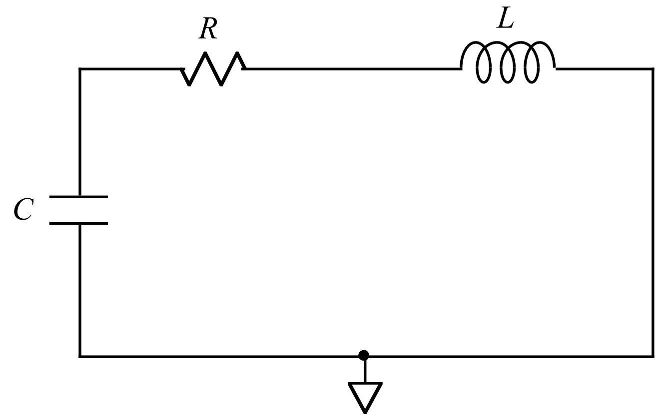
Ex:




$L = 100\ \mu\text{H}$ $\qquad C = 400\ \mu\text{F}$

The above circuit may be used to generate a note for an electronic synthesizer.

a) Find the value of $R$ that will make the circuit oscillate at 440 Hz, (corresponding to an 'A' note). Remember to convert Hz to rad/s.

b) Find the value of the damping factor, $\alpha$, for your circuit.

**SOL'N:** a) The oscillation referred to in the problem must be the oscillation seen in under-damped *RLC* circuits, since over- and critically-damped circuits have only non-oscillatory solutions. The rate of oscillation in under-damped circuits is the damping frequency, $\omega_d$. We convert the damping frequency, $f_d$, given in the problem statement in units of Hertz (or cycles per second) to the damping frequency, $\omega_d$, in units of radians per second:

$$\omega_d = 440\ \text{Hz} \cdot 2\pi \approx 2764.6\ \text{r/s}$$

The formula for the damping frequency, in terms of damping factor, $\alpha$, and resonant frequency, $\omega_o$, is the same for both series and parallel RLC circuits:

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

Using our value for $\omega_d$, we obtain the following equation:

$$\sqrt{\omega_o^2 - \alpha^2} = 2764.6 \quad \text{or} \quad \omega_o^2 - \alpha^2 = 2764.6^2 \approx 7.643\ \text{Mr}^2/\text{s}^2$$

The value of the resonant frequency, $\omega_o$, is always given by the following formula:

$$\omega_o^2 = \frac{1}{LC}$$

or

$$\omega_o^2 = \frac{1}{100\mu \cdot 400\mu} = \left(\frac{1}{200\mu}\right)^2 (\text{r/s})^2 = (5\text{k})^2\ (\text{r/s})^2 = 25\text{ M }(\text{r/s})^2$$

Using the prior equations, we solve for $\alpha^2$:

$$\alpha^2 = \omega_o^2 - 7.643\text{ Mr}^2/\text{s}^2 = 25\text{ M} - 7.643\text{ Mr}^2/\text{s}^2 \approx 17.357\text{ Mr}^2/\text{s}^2$$

or

$$\alpha \approx 4.166\text{ kr/s}$$

For a series RLC circuit, $\alpha$ is one-half the inverse of the L/R time constant:

$$\alpha = \frac{R}{2L}$$

Now we can solve for $R$:

$$R = 2\alpha L = 2(4.166\text{ k})100\mu\ \Omega = 833\text{ m}\Omega$$

**SOL'N:** b) The value of $\alpha$ was found in the solution to part (a):

$$\alpha \approx 4.166\text{ kr/s}$$