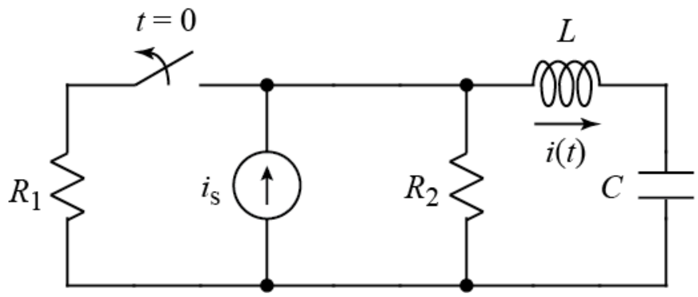
Ex:



After being closed for a long time, the switch opens at $t = 0$.

$L = 1\ \mu H$ $C = 16\ \mu F$ $R_1 = 0.15\ \Omega$ $R_2 = 0.1\ \Omega$

If $i_s = 30$ mA, f ind $i(t)$ for $t > 0$.

SOL'N: We may perform the following initial steps in any order:

1) Find characteristic roots for the parallel RLC circuit (for $t > 0$)
2) Find the final value of $i(t)$ as t -> ∞, which is the A_3 (constant) term in the solution.
3) Find the initial values of energy variables: $i_L(0^+) = i_L(0^-)$ and $v_C(0^+) = v_C(0^-)$

We will perform the steps in the order listed. First, we find the characteristic roots.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

After time $t = 0$, R_1 will be disconnected from the circuit and may be ignored. The characteristic roots are unaffected by source i_s. We have a series RLC circuit for which the value of α is one-half the inverse L/R time constant:

$$\alpha = \frac{R_2}{2L} = \frac{0.1\ \Omega}{2 \cdot (1\mu H)} = 50\ \text{k/s}$$

For both parallel and series RLC circuits, the resonant frequency, ω_o, is the inverse of the square root of the product of L and C:

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{1\ \mu H \cdot 16\ \mu F} = (250k)^2 (r/s)^2$$

Substituting into the equation for the roots yields the following:

$$s_{1,2} = -50 \text{ kr/s} \pm \sqrt{(50\text{k})^2 - (250\text{k})^2} \text{ r/s} \approx -50 \text{ kr/s} \pm j245 \text{ kr/s}$$

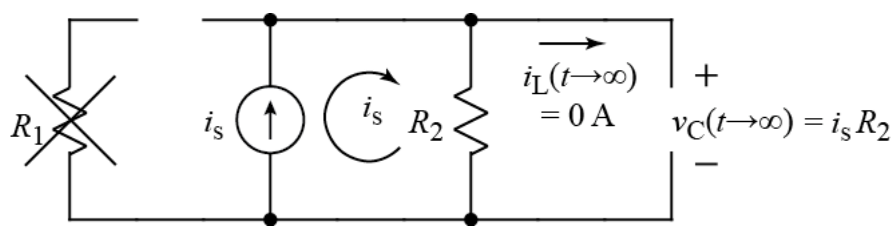
The imaginary part of the roots is the damping frequency, which we may compute by taking the negative of the quantity inside the square root:

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} \approx 245 \text{ kr/s}$$

For complex roots, we use the following form of solution:

$$v(t) = A_1 e^{-\alpha t} \cos(\omega_d t) + A_2 e^{-\alpha t} \sin(\omega_d t) + A_3$$

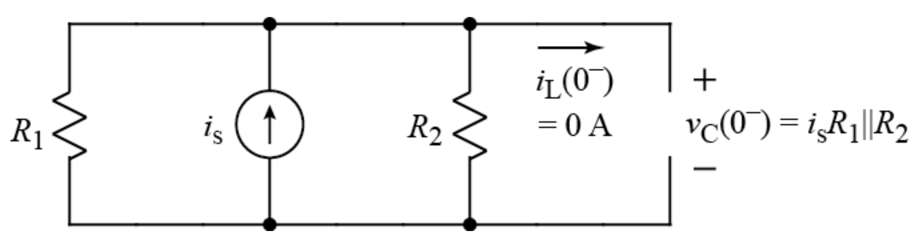
Now we proceed to step 2, where we find the A_3 value from $v(t)$ as t->∞. (The exponential terms decay as t->∞, leaving only A_3.) We assume the circuit values become constant as t->∞, causing the L to act like a wire and the C to act like an open circuit. The switch is also open, detaching R_1 on the left, as shown in the diagram below.



Since the C acts like an open circuit, the current in the L is zero, and we have $A_3 = i(t\text{->}\infty) = 0$ A.

Third, we find the initial values of energy variables: $i_L(0^+) = i_L(0^-)$ and $v_C(0^+) = v_C(0^-)$. At $t = 0^-$, we assume circuit values are constant, causing the L to act like a wire and the C to act like an open circuit. The switch is closed, connecting the parallel RLC to R_1 on the left.

$t = 0^-$:



The C, acting like an open, makes the initial L current zero:

$i_L(0^-) = 0$ A

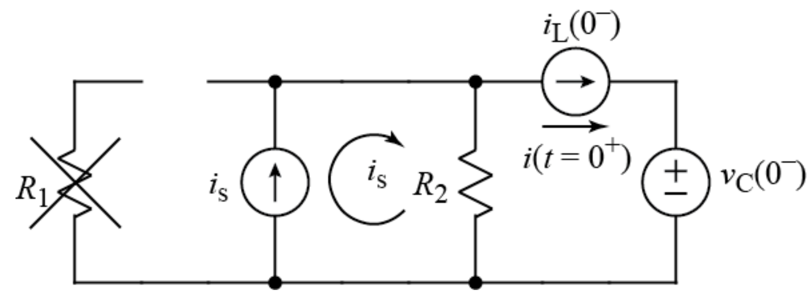
The two R's are in parallel across the voltage source. We may use Ohm's law to find the voltage across the rails. This equals the voltage across the C:

$$v_C(0^+) = v_C(0^-) = i_s \cdot R_1 \parallel R_2 = 30\,\text{mA} \cdot 0.1 \parallel 0.15\,\Omega$$

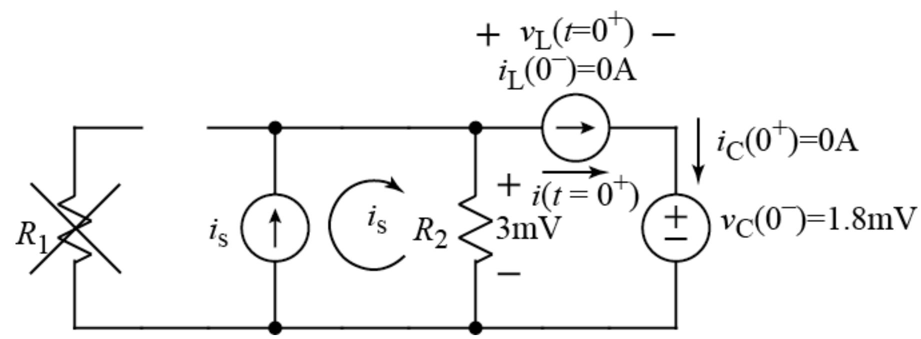
or

$$v_C(0^+) = v_C(0^-) = 30\,\text{mA} \cdot 0.06\,\Omega = 1.8\,\text{mV}$$

At time $t = 0^+$, we treat the C as a voltage source of 1.8 mV and the L as a current source of 0 A. (The switch is also open, removing R_1 from the circuit.)



Because the current in the L is zero, all of i_s flows through R_2. The voltage across R_2, by Ohm's law, is $i_s R_2 = 30\,\text{mA} \cdot 0.1\,\Omega = 3\,\text{mV}$.



We can solve the circuit for any voltage or current at $t = 0^+$. Using Kirchhoff's laws for current sums at nodes, we have a voltage loop on the right that yields the voltage across the L at $t = 0^+$:

$$v_L(0^+) = 3\,\text{mV} - 1.8\,\text{mV} = 1.2\ \text{mV}$$

For the C, no current flows in the L, and so no current flows in the C:

$$i_C(0^+) = 0\,\text{A}$$

Now we are ready to find A_1 and A_2 by matching our symbolic solutions

$$i(0^+) = A_1 + A_3 = A_1$$

and

$$\left.\frac{di(t)}{dt}\right|_{t=0^+} = -\alpha A_1 + \omega_d A_2$$

to circuit values for these quantities. We have already found $i(0^+)$ to be 0 A. Matching this to the symbolic solution for $t = 0^+$, we have the following:

$$A_1 = 0\text{V}$$

For the value of the derivative in the circuit, we first write our variable, ($i(t)$ in this case), in terms of energy (or state) variables, i_L and/or v_C. Here, i is i_L:

$$i(t) = i_L(t)$$

Next, we differentiate, and use the component equations involving d/dt for L and/or C:

$$\left.\frac{d}{dt}i(t)\right|_{t=0^+} = \left.\frac{d}{dt}i_L(t)\right|_{t=0^+} = \frac{v_L(0^+)}{L} = \frac{1.2\text{ mV}}{1\ \mu\text{H}} = 1.2\text{ kA/s}$$

We equate this to the symbolic derivative:

$$\left.\frac{d}{dt}i(t)\right|_{t=0^+} = -\alpha A_1 + \omega_d A_2 = \omega_d A_2 = 1.2\text{ kA/s}$$

or

$$A_2 = \frac{1.2\text{ kA/s}}{\omega_d} \approx \frac{1.2\text{ kA/s}}{245\text{ kr/s}} \approx 4.9\text{ mA/s}$$

Plugging in values gives the solution for $v(t > 0)$:

$$i(t) = 4.9e^{-50\text{ k/s}\cdot t}\sin(245\text{ kr/s}\cdot t)\text{ mA}$$