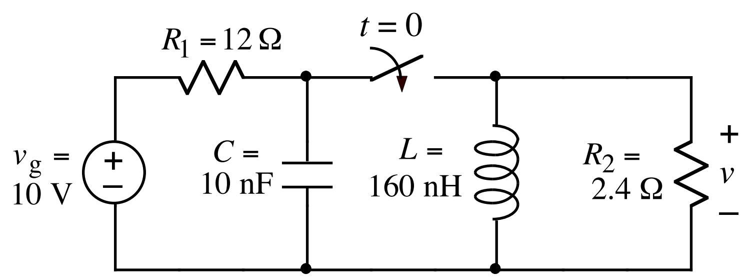
Ex:



After being open for a long time, the switch closes at $t = 0$.

a) State whether $v(t)$ is under-damped, over-damped, or critically-damped.

b) Write a numerical time-domain expression for $v(t)$, $t > 0$, the voltage across R_2 in problem 4. This expression must not contain any complex numbers.

SOL'N: a) After $t=0$, we have a parallel RLC with $R = R_1 \| R_2$. (We could use a Norton equivalent for v_g and R_1 to see that we have $R = R_1 \| R_2$.)

For parallel RLC, we have

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}, \quad \alpha = \frac{1}{2RC}, \quad \omega_0^2 = \frac{1}{LC}$$

where $R = R_1 \| R_2 = 12\,\Omega \| 2.4\,\Omega$

$$= 2.4\,\Omega \cdot 5 \| 1 = 2\,\Omega$$

$$\alpha = \frac{1}{2 \cdot 2\,\Omega \cdot 10\text{nF}} = \frac{1}{40}\text{G/s} = 25\text{ M r/s}$$

$$\omega_0^2 = \frac{1}{160\text{nH} \cdot 10\text{nF}} = \left(\frac{1\text{G}}{40}\right)^2 (\text{r/s})^2 = (25\text{ M})^2 (\text{r/s})^2$$

Since $\alpha^2 = \omega_0^2$, the circuit is critically-damped.

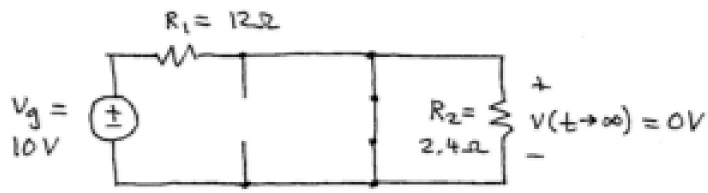
b) For critically-damped circuit, we use

$$v(t) = A_1 e^{-\alpha t} + A_2 t e^{-\alpha t} + A_3$$

where $\alpha = 25\text{ M r/s}$

$A_3 = v(t \to \infty)$.

$t \to \infty$ model: L = wire, C = open, switch closed



Since R_2 is shorted by L, we have

$$A_3 = v(t \to \infty) = 0V$$

Now we match the symbolic and circuit values of $v(0^+)$ and $\left.\frac{dv}{dt}\right|_{t=0^+}$.

symbolic $v(0^+) = A_1 e^{-\alpha \cdot 0^+} + A_2 \cdot 0^+ \cdot e^{-\alpha 0^+}$

makes 2nd term = 0

or

$v(0^+) = A_1$

symbolic $\left.\frac{dv}{dt}\right|_{t=0^+} = \left. -\alpha A_1 e^{-\alpha t} + A_2 e^{-\alpha t} + -\alpha A_2 t e^{-\alpha t} \right|_{t=0^+}$

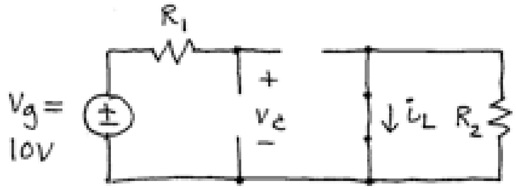
or

$$\left.\frac{dv}{dt}\right|_{t=0^+} = -\alpha A_1 + A_2$$

For the circuit, we start with $t = 0^-$, L = wire, C = open, switch open, and we find i_L and v_C.

We have two separate circuits at $t=0^-$:

$t=0^-$ model:



On the left, v_c charges to 10V.
(There is no current in R_1 and no v-drop across R_1.)

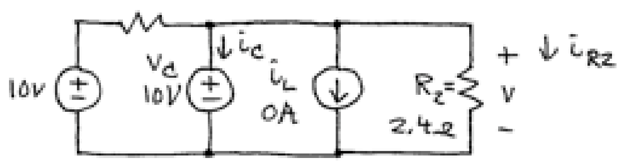
On the right, the current in L must be zero. (L discharges thru R_2.)

$$v_c(0^-) = 10V \qquad i_L(0^-) = 0A$$

$t=0^+$ model: Use sources for v_c and i_L

$$v_c(0^+) = v_c(0^-) = 10V$$
$$i_L(0^+) = i_L(0^-) = 0A \quad \text{(open)}$$

switch closed



Since $\underline{R_2 \text{ is across } C}$, $v(0^+) = v_c(0^+) = 10V$

$$v(0^+) = A_1 = 10\ V$$

For $\left.\frac{dv}{dt}\right|_{t=0^+}$, we have $\frac{dv(t)}{dt} = \frac{dv_c}{dt} = \frac{i_c}{C}$,
(v is across C).

So we want to find $i_c(0^+)$.

Using a current summation for the top node, we have

$$\underbrace{\frac{10V - 10V}{R_1}}_{0A} + i_c(0^+) + \underbrace{i_L(0^+)}_{0A} + \frac{10V}{R_2} = 0A$$

or

$$i_c(0^+) = -\frac{10V}{R_2}$$ (current flows up out of C and down thru R_2)

Thus $$\left.\frac{dv}{dt}\right|_{0^+} = \frac{-\frac{10V}{R_2}}{C} = \frac{-10V}{2.4\Omega \cdot 10nF} = -\frac{1}{2.4} GV/s$$

$$= -\alpha A_1 + A_2 = -25M/s \cdot 10V + A_2$$

We have $A_2 = -\frac{1}{2.4} GV/s + 250MV/s$

$$A_2 \doteq -416.7M + 250MV/s$$

$$A_2 \doteq -166.7 MV/s$$

so

$$v(t) \doteq 10e^{-25Mt} - 167 M/s \cdot te^{-2Mt} \quad V$$