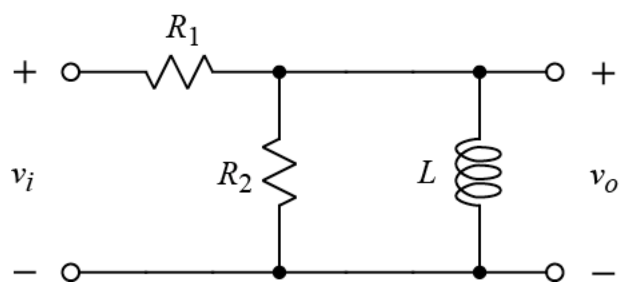
Ex:



$R_1 = 1.5\ \text{k}\Omega$ $\quad R_2 = 3\ \text{k}\Omega$ $\quad L = 40\ \mu\text{H}$

a) Determine the transfer function V_o/V_i. **Hint:** use a Thevenin equivalent on the left side.

b) Plot $|V_o/V_i|$ versus ω.

c) Find the cutoff frequency, ω_c.

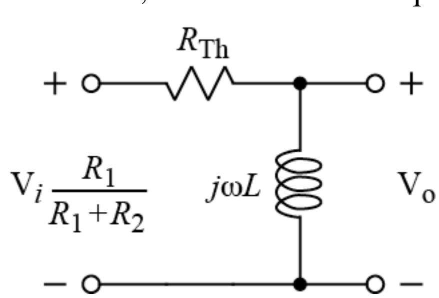
SOL'N: a) We use a Thevenin equivalent, as suggested in the problem, consisting of the input source, v_i, R_1, and R_2. The Thevenin voltage source is obtained by removing the L from the circuit and measuring the voltage across R_2. This leaves a voltage divider:

$$v_{\text{Th}} = v_i \frac{R_2}{R_1 + R_2}$$

To find R_{Th}, we turn off the v_i source, which becomes a wire, and look in from the output terminals of the Thevenin circuit, (i.e., across R_2), and we see resistance $R_1 \| R_2$.

$$R_{\text{Th}} = R_1 \| R_2$$

Our new circuit, converted to the frequency domain, is shown below:



The voltage-divider formula gives the transfer function, starting with the formula for V_o:

$$\mathrm{V_o} = \mathrm{V}_i \frac{R_2}{R_1 + R_2} \frac{j\omega L}{R_{\mathrm{Th}} + j\omega L}$$

Dividing by V_i gives the transfer function. (Note that we do *not* replace V_i with V_{Th}.)

$$H(j\omega) = \frac{\mathrm{V_o}}{\mathrm{V}_i} = \frac{R_2}{R_1 + R_2} \frac{j\omega L}{R_{\mathrm{Th}} + j\omega L}$$

A better form is obtained by dividing top and bottom by $j\omega L$:

$$H(j\omega) = \frac{R_2}{R_1 + R_2} \frac{1}{1 + \dfrac{R}{j\omega L}} = \frac{R_2}{R_1 + R_2} \frac{1}{1 - j\dfrac{R_{\mathrm{Th}}}{\omega L}}$$

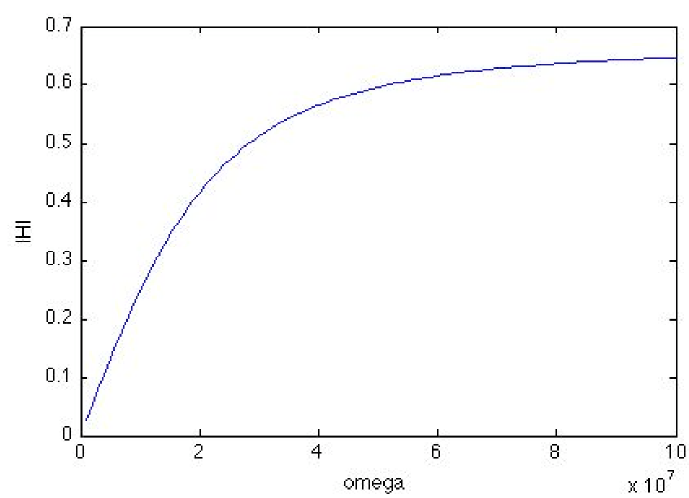
$$= \frac{R_2}{R_1 + R_2} \frac{1}{1 - j\dfrac{1\mathrm{k}}{\omega 40\mu}} = \frac{R_2}{R_1 + R_2} \frac{1}{1 - j\dfrac{25\mathrm{M}}{\omega}}$$

b) The plot is generated with the following Matlab® code.

```
% ECE2260F10_HW3p3soln.m
% Plot of transfer function of RL high-pass filter.

R1 = 1.5e3;   % Ohms
R2 = 3e3;     % Ohms
R_Th = R1 * R2 / (R1 + R2);  % Ohms
L = 40e-6;    % Henry's

omega = 1e6:1e6:100e6;
H = R2/(R1 + R2) * 1 ./ (1 - j*R_Th ./ (omega*L));
plot(omega,abs(H))
xlabel('omega')
ylabel('|H|')
```



c) We find the cutoff frequency by setting the magnitude of the transfer function equal to $1/\sqrt{2}$ times the max value of $|H|$. Here the max value of $|H|$ will be $R_2/(R_1+R_2) = 2/3$.

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}} \max_{\omega} |H(j\omega)| = \frac{1}{\sqrt{2}}\left(\frac{2}{3}\right)$$

The factor of 2/3, however, is also present in the magnitude of H, and we may cancel it out. This leaves us with the following equation:

$$|H(j\omega)| = \left|\frac{1}{1+\dfrac{R_{\mathrm{Th}}}{j\omega L}}\right| = \frac{1}{\sqrt{2}}$$

We observe that $\sqrt{2} = |1 \pm j|$ meaning we can solve for ω_c by setting $R_{\mathrm{Th}}/\omega L$ equal to one:

$$\frac{R_{\mathrm{Th}}}{\omega_c L} = 1$$

or

$$\omega_c = \frac{R_{\mathrm{Th}}}{\mathrm{L}} = \frac{1\mathrm{k}}{40\mu}\,\mathrm{r/s} = 25\mathrm{M\,r/s}$$