HOMEWORK #5 prob 3a solution

EX: Find the inverse Laplace transform for the following expression:

$$F(s) = \frac{12s + 20}{s^2 + 5s}$$

SOL'N: We use partial fractions, meaning we factor the denominator into root terms and find coefficients in the numerators of terms for each root.

$$F(s) = \frac{12s + 20}{s^2 + 5s} = \frac{12s + 20}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5}$$

There are many ways to find A and B. One method is to put the two terms over a common denominator and match coefficients of powers of s. Another method, which we use here, is the "pole cover-up method". In this method, we multiply by a root term and then set s to the root value.

$$A = F(s)s\big|_{s=0} = \left.\frac{12s + 20}{s+5}\right|_{s=0} = \frac{20}{5} = 4$$

Note how multiplying by s caused cancellation of the s in the denominator.

$$B = F(s)(s+5)\big|_{s=-5} = \left.\frac{12s + 20}{s}\right|_{s=-5} = \frac{12(-5) + 20}{-5} = \frac{-40}{-5} = 8$$

Now we have $F(s)$ is a form that is is easily inverse transformed:

$$F(s) = \frac{4}{s} + \frac{8}{s+5}$$

$$\mathcal{L}^{-1}\left\{\frac{4}{s} + \frac{8}{s+5}\right\} = 4u(t) + 8e^{-5t}u(t)$$

NOTE: We multiply the second term by $u(t)$ to remind ourselves that the inverse Laplace transform tells us nothing about a signal before $t = 0$ since the Laplace transform integral includes only time greater than zero.