HOMEWORK #5 prob 3b solution

EX: Find the inverse Laplace transform for the following expression:

$$F(s) = \frac{12s + 66}{s^2 + 16s + 289}$$

SOL'N: The denominator of $F(s)$ has complex roots, which is ascertained by taking half of the coefficient of s (i.e., 16), dividing by two, (to get 8), squaring (to get 64), comparing this value to the constant term (i.e., comparing 64 to 289), and concluding that, since the square is less than the constant term, the roots are complex.

When the roots are complex, we may factor the denominator into complex roots and use the pole cover-up method. This approach, however, forces us to deal with complex numbers. An alternative is to write the denominator in the form found in a decaying sine and cosine, which is what the inverse Laplace transform must be when the denominator roots are complex.

$$F(s) = \frac{12s + 66}{(s+8)^2 + 15^2}$$

Note that we derive the denominator by using half the coefficient of the s term, (i.e., 16/2 = 8) and using a constant term whose square, when added to 8^2 yields the value of the constant term in the denominator, (i.e., 289).

Since we are going to get a decaying sine and/or cosine, we now write the numerator as a coefficient times the numerator (s + a = s + 8) for a decaying cosine plus a coefficient times the numerator (ω = 15):

$$F(s) = A\frac{s+8}{(s+8)^2 + 15^2} + B\frac{15}{(s+8)^2 + 15^2}$$

We now find A and B by setting the numerator of this expression equal to the numerator of the original expression:

$$A(s+8) + B(15) = As + A(8) + B(15) = 12s + 66$$

The coefficient for s must be A = 12, and that means the value of B must be –2 in order to obtain a value of 66 for the constant term.

$$F(s) = 12\frac{s+8}{(s+8)^2+15^2} + (-2)\frac{15}{(s+8)^2+15^2}$$

Now we can inverse Laplace transform these terms.

$$\begin{aligned}
\mathcal{L}^{-1}&\left\{12\frac{s+8}{(s+8)^2+15^2} + (-2)\frac{15}{(s+8)^2+15^2}\right\} \\
&= 12\mathcal{L}^{-1}\left\{\frac{s+8}{(s+8)^2+15^2}\right\} - 2\mathcal{L}^{-1}\left\{\frac{15}{(s+8)^2+15^2}\right\} \\
&= 12e^{-8t}\cos(15t)u(t) - 2e^{-8t}\sin(15t)u(t)
\end{aligned}$$