Ex: Find the inverse Laplace transform for the following expression:

$$F(s)=\frac{s^3+30s^2+229s+340}{s(s^2+18s+85)}$$

SOL'N: We must first address the issue that the numerator is of the same order as the denominator, meaning we need to extract a constant term so that what remains will be in proper form (with numerator of lower order than denominator). We extract the constant by dividing the denominator into the numerator. The result will be the same as taking the ratio of the coefficients of the s^3 terms. Thus, we find that the denominator goes into the numerator exactly once.

$$F(s)=\frac{s^3+30s^2+229s+340}{s(s^2+18s+85)}$$

$$=1+\frac{s^3+30s^2+229s+340-[s(s^2+18s+85)]}{s(s^2+18s+85)}$$

$$=1+\frac{12s^2+144s+340}{s(s^2+18s+85)}$$

The inverse Laplace transform of the 1 is $\delta(t)$. Now we turn our attention to the remainder. We factor the denominator to find the partial fraction terms needed. For the quadratic part of the denominator, we use a form for the complex roots that readily inverse transforms to a decaying cos() or sin().

$$\frac{12s^2+144s+340}{s(s^2+18s+85)}=\frac{A}{s}+\frac{B(s+9)}{(s+9)^2+2^2}+\frac{C(2)}{(s+9)^2+2^2}$$

Note that we find the representation for the quadratic in the denominator by dividing the $18s$ term's coefficient by two. Also, $\frac{B(s+9)}{(s+9)^2+2^2}$ corresponds to the form $B\frac{(s+a)}{(s+a)^2+\omega^2}$, which inverse transforms to $Be^{-at}\cos(\omega t)$ or $Be^{-9t}\cos(2t)$. Likewise, $\frac{C(2)}{(s+9)^2+2^2}$ corresponds to the form $C\dfrac{\omega}{(s+a)^2+\omega^2}$, which inverse transforms to $Ce^{-at}\sin(\omega t)$ or $Ce^{-9t}\sin(2t)$.

Now we find *A* by the pole cover-up method:

$$A = sF(s)\Big|_{s=0} = \left.\frac{12s^2+144s+340}{(s^2+18s+85)}\right|_{s=0} = \frac{340}{85} = 4$$

To simplify the process of finding *B* and *C*, we may subtract the $\dfrac{A}{s}=\dfrac{4}{s}$ term from *F*(*s*).

$$\begin{aligned} F(s)-\frac{4}{s} &= \frac{12s^2+144s+340}{s(s^2+18s+85)} - \frac{4}{s}\cdot\frac{(s^2+18s+85)}{(s^2+18s+85)} \\ &= \frac{8s^2+72s}{s(s^2+18s+85)} \\ &= \frac{8s+72}{s^2+18s+85} \\ &= \frac{8s+72}{(s+9)^2+2^2} = B\frac{s+9}{(s+9)^2+2^2} + C\frac{2}{(s+9)^2+2^2} \end{aligned}$$

Matching up coefficients in the numerator, we have that

$$Bs = 8s \quad \text{and} \quad B(9)+C(2)=72$$

We find that $B = 8$ and $C = 0$. Thus, we have the following result:

$$F(s) = \frac{s^3+30s^2+229s+340}{s(s^2+18s+85)} = 1+\frac{4}{s}+\frac{8(s+9)}{(s^2+9^2)+2^2}$$

Taking the inverse transform yields the following answer:

$$f(t) = \delta(t)+4u(t)+8e^{-9t}\cos(2t)u(t)$$