HOMEWORK #6 prob 1 solution

EX:

a) Find $\mathcal{L}\{\delta(t-4)u(t-4)+t\cos(9t)\}$.

b) Find $v(t)$ if $V(s)=\dfrac{16}{s^2+10s+25}$.

c) Find $\lim_{t\to\infty} v(t)$ if $V(s)=\dfrac{10s^2+4}{s^3+s^2+s}$.

d) Plot the poles and zeros of $V(s)$ in the s plane.

$$V(s)=\frac{s^2-s-6}{s^3+6s^2+34s}$$

SOL'N: a) We use the delay transform for the first part of the expression and the identity for multiplication by t for the second part of the expression:

The delay identity is as follows:

$$\mathcal{L}\{f(t-a)u(t-a)\}=e^{-as}\mathcal{L}\{f(t)\}$$

Here, we have the following result:

$$\mathcal{L}\{\delta(t-4)u(t-4)\}=e^{-4s}\mathcal{L}\{\delta(t)\}=e^{-4s}$$

NOTE: The multiplication by the delayed step function actually has no effect on the delayed delta function.

Next, we apply the following identity for multiplication by t:

$$\mathcal{L}\{tf(t)\}=-\frac{d}{ds}F(s)$$

Here, we have the following result:

$$\mathcal{L}\{t\cos(9t)\}=-\frac{d}{ds}\frac{s}{s^2+9^2}=\frac{-1}{s^2+9^2}+\frac{s(2s)}{[s^2+9^2]^2}$$

We sum the results for the final answer:

$$\mathcal{L}\{\delta(t-4)u(t-4)+t\cos(9t)\}=e^{-4s}-\frac{1}{s^2+9^2}+\frac{2s^2}{[s^2+9^2]^2}$$

b) We first factor the denominator.

$$s^2+10s+25=(s+5)^2$$

We can take the inverse transform immediately for this form of denominator:

$$v(t) = \mathcal{L}^{-1}\left\{\frac{16}{(s+5)^2}\right\} = \left[16te^{-5t}\right]u(t)$$

NOTE: We multiply by $u(t)$ to suggest that nothing is known about the signal before time zero.

c) We use the initial value theorem:

$$\lim_{t\to\infty} v(t) = \lim_{s\to 0} sV(s)$$

or

$$\lim_{t\to\infty} v(t) = \lim_{s\to 0} s\frac{10s^2+4}{s^3+s^2+s}$$

We cancel a factor of s from the top and bottom and substitute $s = 0$.

$$\lim_{t\to\infty} v(t) = \lim_{s\to 0} \frac{10s^2+4}{s^2+s+1} = 4$$

d) The poles are roots of the denominator, and the zeros are the roots of the numerator.

$$V(s) = \frac{s^2-s-6}{s^3+6s^2+34s} = \frac{(s-3)(s+2)}{s(s+3+j5)(s+3-j5)}$$

The zeros are 3 and –2. The poles are 0, –3–j5, –3+j5.

We denote the poles with x's and the zeros with o's in the complex s-plane: