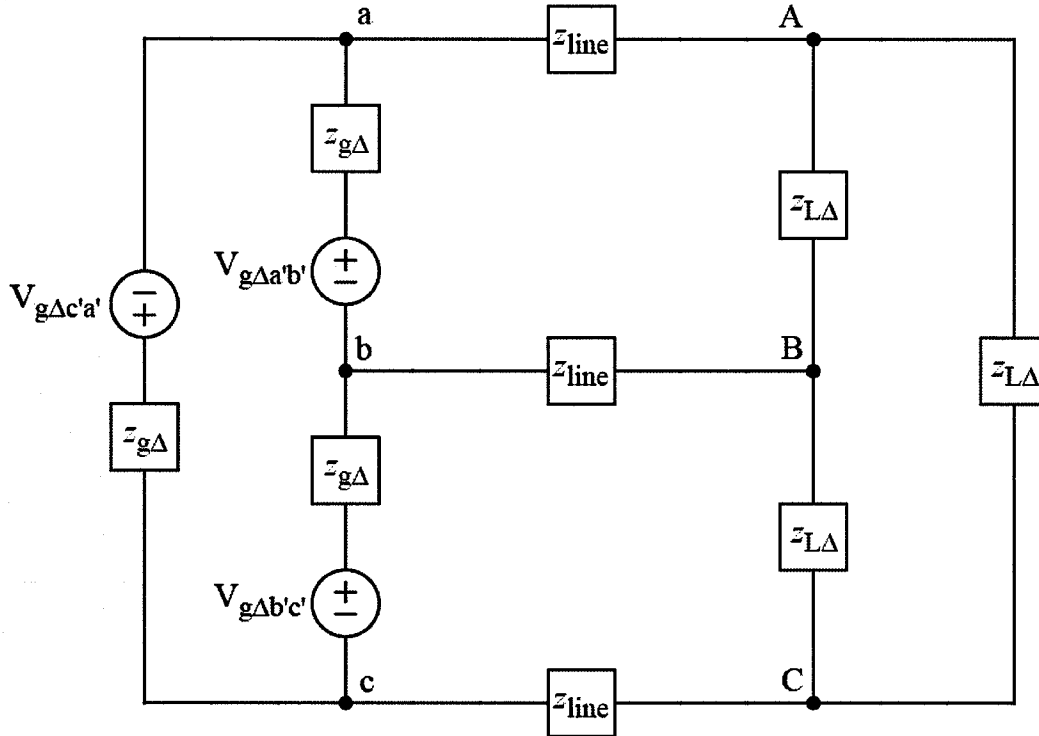


Ex:



$V_{g\Delta a'b'} = 265 \angle 0^\circ \text{ V}$ $z_{g\Delta} = 8.3 \Omega$ resistor in series with $5/6 \mu\text{F}$ capacitor

$V_{g\Delta b'c'} = 265 \angle -120^\circ \text{ V}$ $z_{\text{line}} = 1 \Omega$ resistor in series with 700 nH inductor

$V_{g\Delta c'a'} = 265 \angle +120^\circ \text{ V}$ $z_{L\Delta} = 4 \Omega$ resistor in series with $10/9 \mu\text{F}$ capacitor

- Draw the single-phase equivalent circuit. Note: $\omega = 1 \text{ Mr/s}$.
- Calculate I_{bB} .
- Calculate V_{AB} .
- Write a numerical time-domain expression for $v_{AB}(t)$.

sol'n: a) We convert the source and the load to Y configurations.

$$z_{gY} = \frac{z_{g\Delta}}{3} \quad \text{and} \quad z_{LY} = \frac{z_{L\Delta}}{3}$$

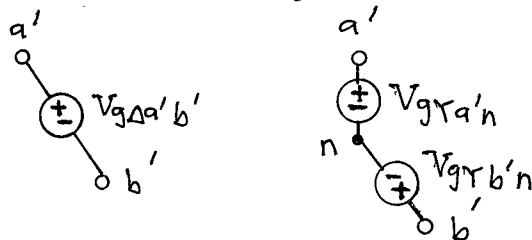
$$\text{where } z_{g\Delta} = 8.3\Omega - \frac{1\Omega}{j1M \cdot \frac{5\mu}{6}}, \quad z_{L\Delta} = \frac{4\Omega - 1\Omega}{j1M \cdot \frac{10\mu}{9}}$$

$$" = 8.3\Omega - j\frac{6}{5}\Omega, \quad " = 4\Omega - j0.9\Omega$$

$$\text{So } z_{gY} = \frac{8.3}{3}\Omega - j0.4\Omega, \quad z_{LY} = \frac{4}{3}\Omega - j0.3\Omega$$

For the V source, we have

$$V_{g\Delta a'b'} = V_{gYa'n} - V_{gYb'n}$$



$$\text{We know } V_{gYb'n} = V_{gYa'n} \cdot 1\angle -120^\circ$$

because the phases shift by -120° as we proceed in a clockwise direction.

We create a diagram showing how $V_{g\Delta a'b'}$ is computed from $V_{gYa'n}$ and invert the resulting eq'n in order to compute $V_{gYa'n}$ from $V_{g\Delta a'b'}$.

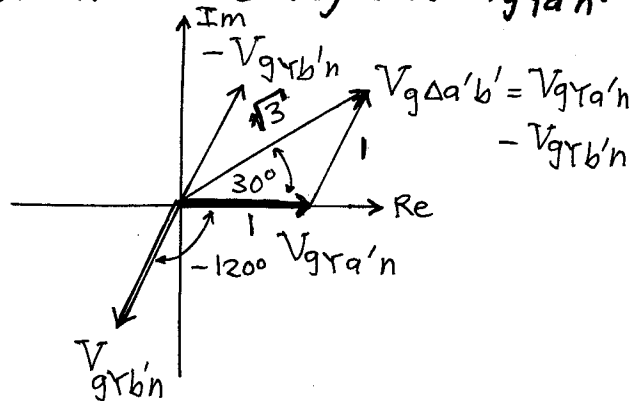
Note: The diagram would be too difficult to draw if we tried to start with $z_{g\Delta}$ and find z_{gY} .

In eq'n form, we have

$$\begin{aligned}
 V_{g\Delta a'b'} &= V_{gYa'n} - V_{gYa'n} \angle -120^\circ \\
 &= V_{gYa'n} (1 - \angle -120^\circ) \\
 &= V_{gYa'n} \left[1 - \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) \right] \\
 &= V_{gYa'n} \left(\frac{3}{2} + j\frac{\sqrt{3}}{2} \right) \\
 V_{g\Delta a'b} &= V_{gYa'n} \sqrt{3} \angle +30^\circ
 \end{aligned}$$

Using a phasor diagram, we start with $V_{gYa'n}$ drawn on the real

axis. This gives the correct final answer because our final eq'n tells us how to get from $V_{gYa'n}$ to $V_{g\Delta a'b'}$ and doesn't depend on the absolute angle of $V_{gYa'n}$.

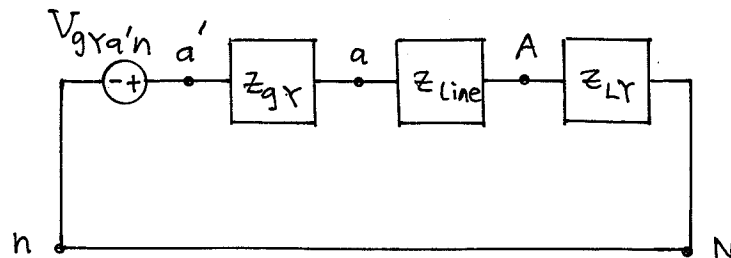


we have $V_{g\Delta a'b'} = V_{gYa'n} \cdot \sqrt{3} \angle +30^\circ$

and $V_{gYa'n} = V_{g\Delta a'b'} \cdot \frac{1}{\sqrt{3}} \angle -30^\circ$
 $= 265 \angle 0^\circ \text{ V} \cdot \frac{1}{\sqrt{3}} \angle -30^\circ$

$V_{gYa'n} = 153 \angle -30^\circ \text{ V}$

Single-phase equivalent model:



$$V_{gY a'n} = 153 \angle -30^\circ \text{ V} \quad z_{gY} = \frac{8.3}{3} - j0.4 \Omega$$

$$z_{Line} = 1 + j1M(700n) \Omega$$

$$" = 1 + j0.7 \Omega$$

$$z_{LY} = \frac{4}{3} - j0.3 \Omega$$

b) $I_{bB} = I_{aA}$ shifted by -120°

$$I_{aA} = \frac{V_{gY a'n}}{z_{tot}} = \frac{V_{gY a'n}}{z_{gY} + z_{Line} + z_{LY}}$$

(from single-phase model)

$$\text{So } I_{aA} = \frac{153 \angle -30^\circ \text{ V}}{\frac{8.3}{3} - j0.4 + 1 + j0.7 + \frac{4}{3} - j0.3 \Omega}$$

$$= \frac{153 \angle -30^\circ \text{ V}}{\frac{15.3}{3} \Omega}$$

$$= 30 \angle -30^\circ$$

$$I_{bB} = 30 \angle -30^\circ \text{ V} (1 \angle -120^\circ)$$

$$I_{bB} \doteq 30 \angle -150^\circ \text{ A}$$

c) The relationship of V_{AB} to V_{AN}

will be the same as that of
 $V_{g\Delta a'b'}$ to $V_{gYa'n}$:

$$V_{AB} = V_{AN} \cdot \sqrt{3} \angle 30^\circ$$

We find V_{AN} using the single-phase model.

$$V_{AN} = V_{gYa'n} \cdot \frac{Z_{LY}}{Z_{tot}}$$

$$\doteq 153 \angle -30^\circ \text{ V} \frac{\frac{4}{3} - j0.3 \ \Omega}{\frac{15.3}{3} \ \Omega}$$

$$V_{AN} \doteq 30 \angle -30^\circ \text{ V} \left(\frac{4}{3} - j0.3 \right)$$

$$\text{So } V_{AB} \doteq 30 \angle -30^\circ \text{ V} \left(\frac{4}{3} - j0.3 \right) \cdot \sqrt{3} \angle 30^\circ$$

$$\doteq 30\sqrt{3} \left(\frac{4}{3} - j0.3 \right) \text{ V}$$

$$V_{AB} \doteq 40\sqrt{3} - j9\sqrt{3} \text{ V} \quad \text{rect form}$$

$$\doteq 41\sqrt{3} \angle -12.4^\circ \text{ V}$$

$$V_{AB} \doteq 71 \angle -12.4^\circ \text{ V} \quad \text{polar form}$$

d) Taking the inverse phasor of V_{AB} , we have

$$v_{AB}(t) \doteq 71 \cos(1Mt - 12.4^\circ) \text{ V.}$$