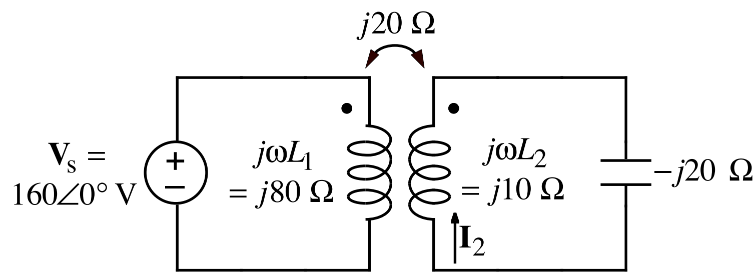
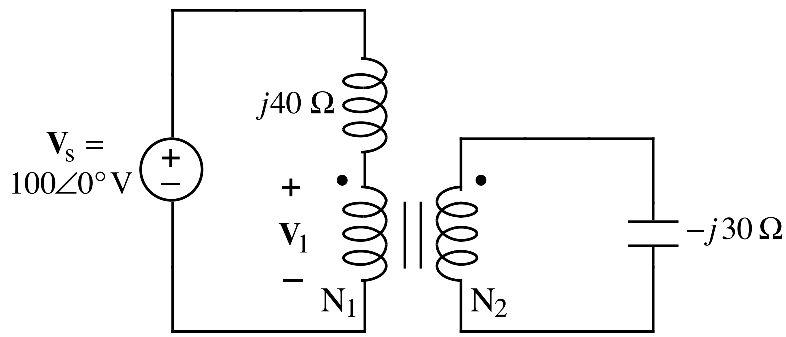
HOMEWORK #8 probs 4 and 5 solution

Ex:



a) Calculate the numerical value of phasor current, $\mathbf{I}_2$, flowing upward in the right coil of the transformer in the above circuit. Note: the transformer is linear.



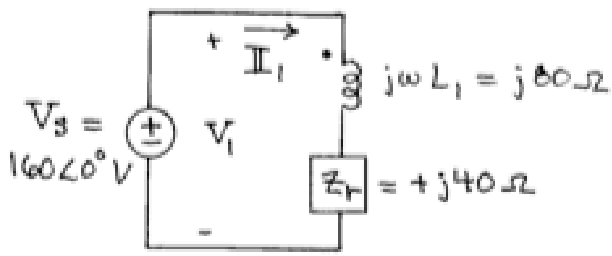
b) Given $\mathbf{V}_1 = 150\angle 0°$ V find the turns ratio, N_1/N_2, for the transformer in the above circuit. Note: the transformer is ideal.

sol'n: a) We first use reflected impedance to find $\mathbb{I}_1$ (current in primary). Then we use the transformer eqn's to find $\mathbb{I}_2$.

Reflected z: $$z_r = \frac{(\omega M)^2}{z_{\text{sec tot}}} = \frac{20^2 \quad \Omega^2}{j10 - j20\,\Omega}$$

$$z_r = \frac{400}{-j10} = j\,40\,\Omega$$

Model of primary:



$$\mathbb{I}_1 = \frac{V_g}{j\omega L_1 + z_r} = \frac{160\angle 0^\circ V}{j80 + j40\,\Omega}$$

or

$$\mathbb{I}_1 = -j\frac{4}{3}\,A \quad \text{or} \quad \frac{4}{3}\angle -90^\circ A$$

Transformer eqn's:

$$V_1 = (R_1 + j\omega L_1)\mathbb{I}_1 - j\omega M\mathbb{I}_2$$

$$V_2 = +j\omega M\,\mathbb{I}_1 - (R_2 + j\omega L_2)\mathbb{I}_2$$

where $R_1 = 0\,\Omega$, $R_2 = 0\,\Omega$.

We observe that $V_1 = V_s$, and we can use the first transformer eqn to find $\mathbb{I}_2$.

$$\mathbf{V}_s = j\omega L_1 \mathbf{I}_1 - j\omega M \mathbf{I}_2$$

or

$$\mathbf{I}_2 = \frac{j\omega L_1 \mathbf{I}_1 - \mathbf{V}_s}{j\omega M}$$

or

$$\mathbf{I}_2 = \frac{j\,80\,\Omega\left(-j\frac{4}{3}\right)\,A \quad - 160\angle 0^\circ\,V}{j\,20\,\Omega}$$

or

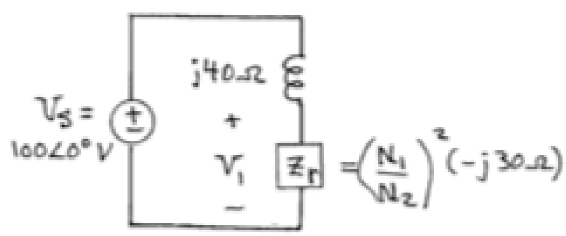
$$\mathbf{I}_2 = \frac{\frac{320}{3} - \frac{480}{3}\ V}{j\,20\,\Omega}$$

or

$$\mathbf{I}_2 = -\frac{160}{j60}\ A = j\,\frac{8}{3}\,A$$

b) We replace the transformer with the reflected impedance $Z_r = \left(\frac{N_1}{N_2}\right) Z_L$

where $Z_L = -j\,30\,\Omega$:



Using a voltage-divider, we have

$$\mathbf{V}_1 = \mathbf{V}_s \frac{Z_r}{j40\Omega + Z_r}$$

or

$$\mathbf{V}_1 (j40\Omega + Z_r) = \mathbf{V}_s Z_r$$

or

$$\mathbf{V}_1 (j40\Omega) = (\mathbf{V}_s - \mathbf{V}_1)\, Z_r$$

or

$$Z_r = \frac{\mathbf{V}_1 (j40\Omega)}{\mathbf{V}_s - \mathbf{V}_1} = \frac{150(j40\Omega)}{100 - 150} = -j\,120\,\Omega$$

But $Z_r = (N_1/N_2)(-j30\Omega)$, so $(N_1/N_2)^2 = 4$, $\frac{N_1}{N_2} = 2$.