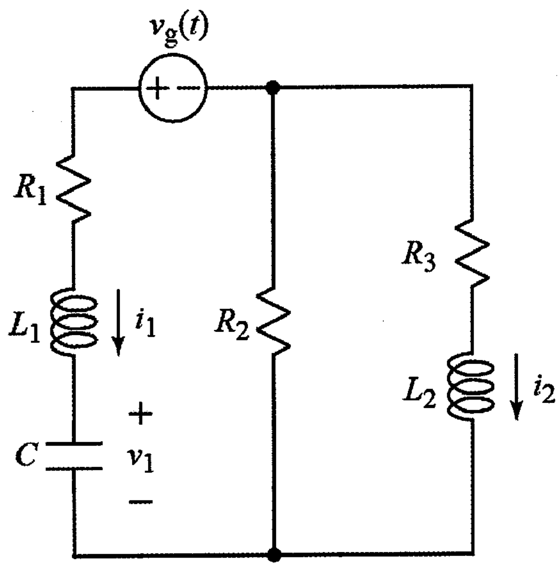
Ex:



At $t = 0$, $v_g(t)$ switches instantly from $-v_o$ to v_o.

a) Write the state-variable equations for the circuit in terms of the state vector:

$$\vec{x} = \begin{bmatrix} i_1 \\ i_2 \\ v_1 \end{bmatrix}$$

b) Evaluate the state vector at $t = 0^+$.

sol'n: a) On the left of each eq'n, we have the first derivatives of energy vars. We equate these derivatives with non-derivatives via the component eq'ns for L and C.

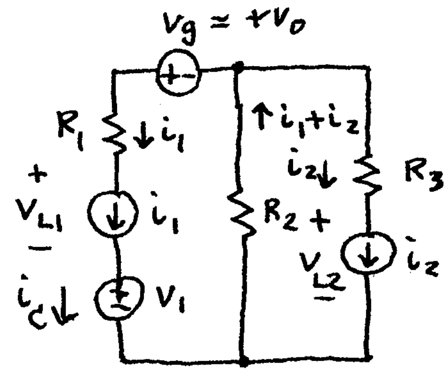
$$di_1/dt = v_{L1}/L_1$$

$$di_2/dt = v_{L2}/L_2$$

$$dv_1/dt = i_C/C$$

Now we find the eq'ns for expressing v_{L1}, v_{L2}, and i_C in terms of energy vars i_1, i_2, and v_1.

We may replace the L's and C with sources labeled i_1, i_2, and v_C.



From Kirchhoff's laws, we have current $i_1 + i_2$ flowing up in the center branch, since i_1 and i_2 are flowing down in the outer branches.

For v_{L1}, we may use a v-loop on the left: (proceed clockwise from lower left)

$$v_1 + v_{L1} + i_1 R_1 - v_0 - (i_1 + i_2) R_2 = 0V$$

or

$$v_{L1} = v_0 + (i_1 + i_2) R_2 - v_1 - i_1 R_1$$

For v_{L2}, we may use a v-loop on the right:

$$-(i_1 + i_2) R_2 - i_2 R_3 - v_{L2} = 0V$$

$$v_{L2} = -(i_1 + i_2) R_2 - i_2 R_3$$

Finally, we have $i_c = i_1$.

$$\frac{di_1}{dt} = \frac{v_0 + (i_1 + i_2)R_2 - v_1 - i_1 R_1}{L_1}$$

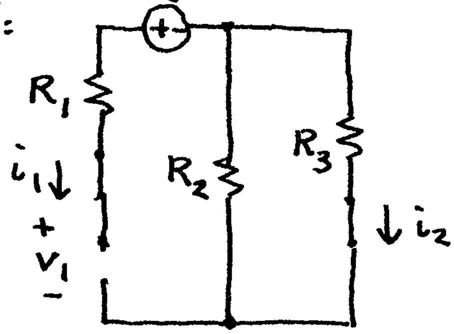
$$\frac{di_2}{dt} = \frac{-(i_1 + i_2)R_2 - i_2 R_3}{L_2}$$

$$\frac{dv_1}{dt} = \frac{i_1}{C}$$

b) The state vector, being energy vars, has the same value at $t = 0^+$ as at $t = 0^-$.

At $t = 0^-$, we may model the L as a wire and the C as an open.

$t = 0^-$:



$i_1(0^-) = 0\,A$ since the C in series with the L is an open.

$i_2(0^-) = 0\,A$ since there is no pwr source connected to $R_2 /\!/ R_3$.

$v_1(0^-) = -v_0$ since no current flows in R_1 or $R_2 /\!/ R_3$ and there is no V-drop across the R's.

$$\begin{bmatrix} i_1(0^+) \\ i_2(0^+) \\ v_1(0^+) \end{bmatrix} = \begin{bmatrix} 0\ A \\ 0\ A \\ -V_0 \end{bmatrix}$$