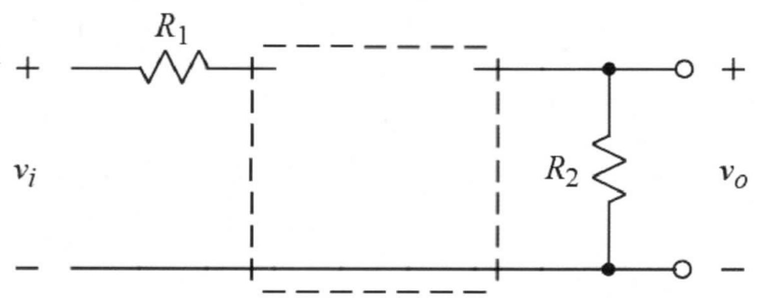
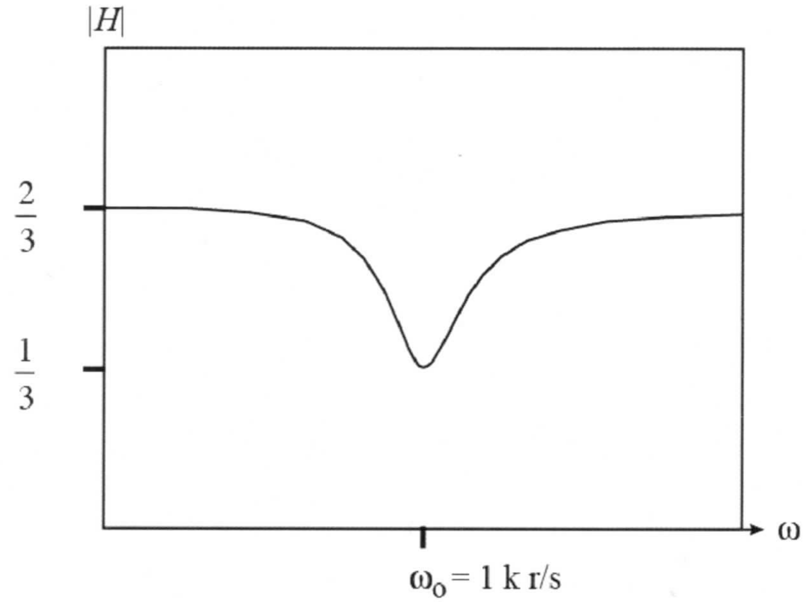
Ex:





Given the resistors connected as shown with the following values,

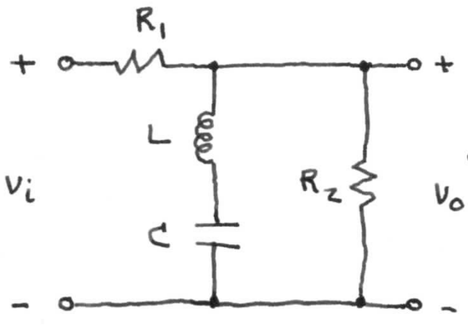
$R_1 = 1\text{ k}\Omega$ $\qquad$ $R_2 = 2\text{ k}\Omega$

and using not more than one each *R*, *L*, and *C* in the dashed-line box, design a circuit to go in the dashed-line box that will produce the **band-reject** $|H(j\omega)|$ vs. ω shown above. That is:

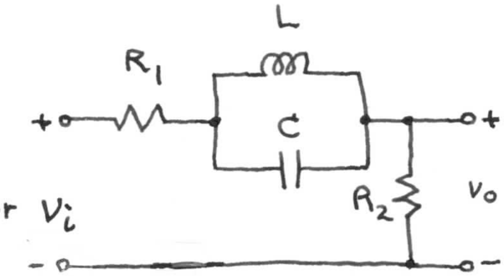
$$\min_{\omega} |H(j\omega)| = \tfrac{1}{3} \text{ and occurs at } \omega_0 = 1 \text{ k r/s}$$

$$|H(j\omega)| = \frac{2}{3} \text{ at } \omega = 0 \qquad \text{and} \qquad \lim_{\omega \to \infty} |H(j\omega)| = \frac{2}{3}$$

sol'n: To create the dip in $|H|$ at $\omega_0 = 1\text{kr/s}$, we may use one of two methods: a vertical LC that shorts out at ω_0, or a horizontal LC that becomes an open circuit at ω_0. The vertical LC would be in series. The horizontal LC would be in parallel.



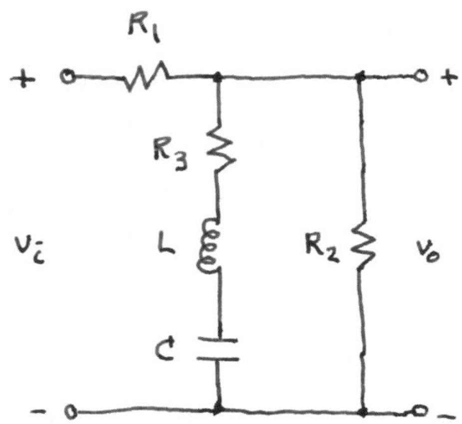
or

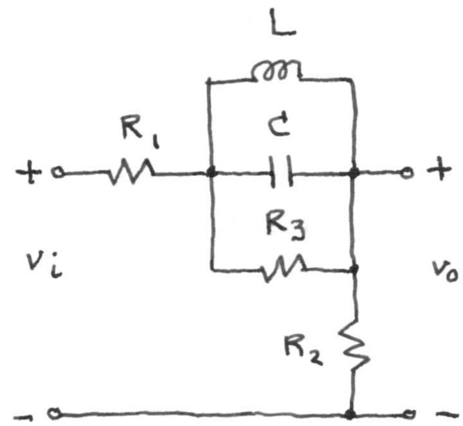


I'

II'

Circuit I' would yield a gain of zero at ω_0, so we need to add an R in series with L and C. Circuit II' would also yield a gain of zero at ω_0, so we need to add an R in parallel with L and C. Thus, we have the two possible circuits shown below.



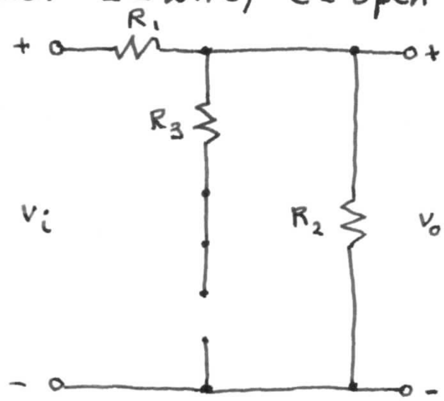


I

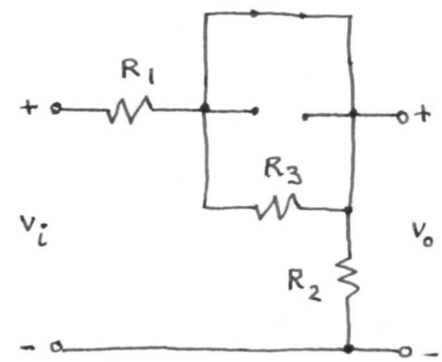
II

We now consider the behavior of each circuit at $\omega = 0$, $\omega = \omega_0$, and $\omega \to \infty$.

$\omega = 0$: L = wire, C = open



I

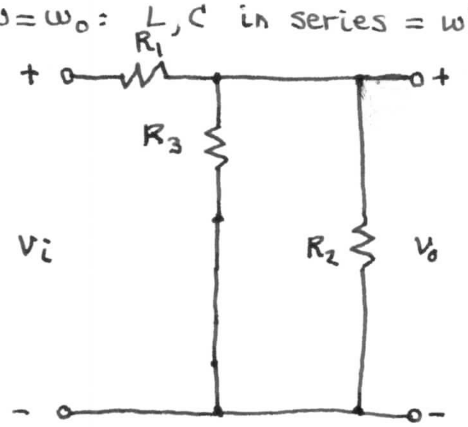


II

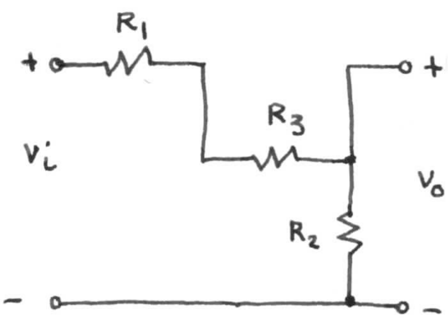
$$H(j0) = \frac{V_o}{V_i} = \frac{R_2}{R_1+R_2} = \frac{2k\Omega}{1k\Omega + 2k\Omega} = \frac{2}{3} \quad \text{for both circuits}$$

Note: R_3 is bypassed by L in circuit II.

$\omega = \omega_o$: L, C in series = wire, L, C in parallel = open



I



II

$$\text{I}: H(j0) = \frac{V_o}{V_i} = \frac{R_2 \| R_3}{R_1 + R_2 \| R_3} \qquad \text{II}: H(j0) = \frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2 + R_3}$$

We want $|H(j0)| = \frac{1}{3}$ @ $\omega = \omega_o$.

$$\text{I}: \frac{R_2 \| R_3}{R_1 + R_2 \| R_3} = \frac{1}{3} \qquad \text{II}: \frac{R_2}{R_1 + R_2 + R_3} = \frac{1}{3}$$

$$3(R_2 \| R_3) = R_1 + R_2 \| R_3 \qquad 3R_2 = R_1 + R_2 + R_3$$

$$R_2 \| R_3 = \frac{R_1}{2} = \frac{1k\Omega}{2} = \frac{1}{2}k\Omega \qquad R_3 = 2\underset{2k\Omega}{\underset{\shortparallel}{R_2}} - \underset{1k\Omega}{\underset{\shortparallel}{R_1}} = 3k\Omega$$

An interesting way to calculate R_3 is to use the formula for parallel resistance in terms of conductance, $(1/R)$.

$$\frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} = \frac{1\,k\Omega}{2}$$

or

$$\frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{\frac{1}{2}k\Omega}$$

or

$$\frac{1}{R_3} = \frac{1}{\frac{1}{2}k\Omega} - \frac{1}{R_2}$$

or

$$R_3 = \frac{1}{\frac{1}{\frac{1}{2}k\Omega} + \frac{1}{-R_2}}$$

or

$$R_3 = \tfrac{1}{2}k\Omega \,\|\, (-R_2) = \tfrac{1}{2}k\Omega \,\|\, -2k\Omega$$

$$" = 1k\Omega \cdot \tfrac{1}{2} \,\|\, -2$$

$$" = 1k\Omega \cdot \frac{\frac{1}{2}(-2)}{\frac{1}{2} - 2}$$

$$" = 1k\Omega \cdot \frac{-1}{-\frac{3}{2}}$$

$$R_3 = 1k\Omega \cdot \frac{2}{3} \approx 667\,\Omega$$

For L and C, we have $w_0 = \frac{1}{\sqrt{LC}} = 1k\,r/s$.

Thus, $LC = \frac{1}{w_0^2} = \frac{1}{(1k\,r/s)^2} = 1\mu\ (r/s)^2$.

Any $LC = 1\mu$ will work.

For example $L = 10\,mH$ and $C = 100\,\mu F$ is one practical solution.