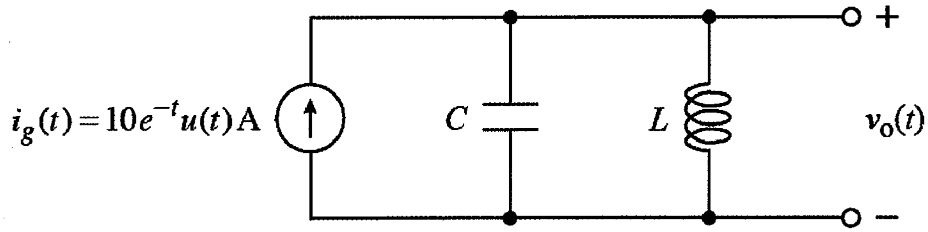
**Ex:**




**Note:** The initial conditions for $C$ and $L$ are zero.

$C = 1$ F $\qquad\qquad L = 250$ mH

a) Write the Laplace transform $I_g(s)$ of $i_g(t)$.

b) Write the Laplace transform $V_o(s)$ of $v_o(t)$. Write your answer as a ratio of polynomials in $s$ with numerical coefficients.

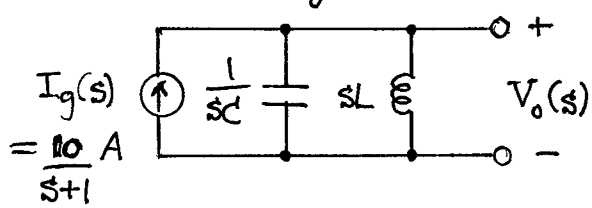
c) Write a numerical time-domain expression for $v_o(t)$ where $t \geq 0$.

SOL'N: a)

$$\mathcal{L}\{i_g(t)\} = \mathcal{L}\{10e^{-t}u(t)\} \text{ A}$$

$$\mathcal{L}\{i_g(t)\} = \frac{10}{s+1} \text{ A}$$

b) Since initial conditions are zero, we may proceed directly to the s-domain circuit diagram.




$$V_o(s) = I_g(s) \cdot \frac{1}{sC} \parallel sL$$

$$V_o(s) = \frac{10}{s+1} \text{ A} \cdot \frac{1}{sC} \parallel sL$$

$$V_o(s) = \frac{10}{s+1} \text{ A} \cdot \frac{L/C}{sL + 1/sC}$$

$$V_o(s) = \frac{10}{s+1} \cdot \frac{250m/1}{s(250m) + \frac{1}{s}} \cdot \frac{s}{s}$$

$$= \frac{10}{s+1} \cdot \frac{s/4}{s^2/4 + 1} \cdot \frac{4}{4}$$

$$V_o(s) = \frac{10s}{(s+1)(s^2+4)}$$

c) $v_o(t) = \mathcal{L}^{-1}\{V_o(s)\}$

Use partial fractions

$$V_o(s) = \frac{A}{s+1} + \frac{Bs+C}{s^2+2^2}$$

$$= \frac{A(s^2+2^2)+(Bs+C)(s+1)}{(s+1)(s^2+2^2)}$$

$$= \frac{As^2 + 4A + Bs^2 + (B+C)s + C}{(s+1)(s^2+2^2)}$$

$$= \frac{(A+B)s^2 + (B+C)s + 4A+C}{(s+1)(s^2+2^2)}$$

$$= \frac{10s}{(s+1)(s^2+2^2)}$$

Matching coefficients of powers of $s$, we have

$A+B=0, \quad B+C=10, \quad 4A+C=0$

$A=-B \Rightarrow -A+C=10, \quad 4A+C=0$

$C=10+A \Rightarrow 4A+(10+A)=0$

$5A+10=0$

$A=-2, \quad B=2, \quad C=8$

So $$V_o(s) = \frac{-2}{s+1} + \frac{2s+8}{s^2+2^2}$$

$$V_o(s) = -\frac{2}{s+1} + 2\cdot\frac{s}{s^2+2^2} + 4\cdot\frac{2}{s^2+2^2}$$

$$v_o(t) = \left[-2e^{-t} + 2\cos(2t) + 4\sin(2t)\right]u(t)\ \text{V}$$