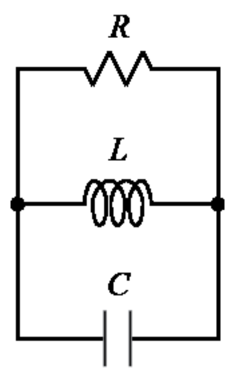
HOMEWORK #1 prob 1 solution

Ex:



$R = 0.5\ \Omega$ $L = 1.5\ \mu\text{H}$ $C = 1.5\ \mu\text{F}$

a) Find the characteristic roots, s_1 and s_2, for the above circuit.

b) Is the circuit over-damped, critically-damped, or under-damped? Explain your answer.

c) If the L and C values in the circuit are decreased by a factor of two, (and R remains the same), will the circuit be over-damped, critically-damped, or under-damped? Justify your answer with calculations.

SOL'N: a) For a parallel RLC circuit (or a series RLC circuit), we have the following formula for the characteristic roots:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

For a parallel RLC circuit, the value of α is one-half the inverse RC time constant:

$$\alpha = \frac{1}{2RC} = \frac{1}{2(0.5\Omega)1.5\mu\text{F}} = \frac{1\text{ M/s}}{1.5} \approx 667\text{ k/s}$$

For both parallel and series RLC circuits, the resonant frequency, ω_o, is the inverse of the square root of the product of L and C:

$$\omega_o = \frac{1}{\sqrt{LC}}$$

or

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{1.5\ \mu\text{H} \cdot 1.5\ \mu\text{F}} = \left(\frac{1}{1.5\ \mu}\text{ r/s}\right)^2 \approx (667\text{ kr/s})^2$$

We find that, since $\alpha = \omega_o$, the roots are equal:

$$s_{1,2} \approx -667 \text{ kr/s} \pm \sqrt{(667 \text{ kr/s})^2 - (667 \text{ kr/s})^2} = -667 \text{ kr/s}$$

SOL'N: b) When the two roots are the same, the circuit is critically-damped.

SOL'N: c) If both L and C decrease by a factor of 2, the resonant frequency, ω_o, will increase by a factor of $\sqrt{2} \cdot \sqrt{2} = 2$.

$$\omega_o = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \omega_o = \frac{1}{\sqrt{LC}} = \frac{500 \text{ kr/s}}{\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}} \approx 1.33 \text{ Mr/s}$$

Because C decreases by a factor of 2, the value of α will also increase by a factor of 2:

$$\alpha = \frac{667 \text{ kr/s}}{\frac{1}{2}} \approx 1.33 \text{Mr/s}$$

Thus, it is still the case that $\alpha = \omega_o$, the two roots are the same, and the circuit is critically-damped.