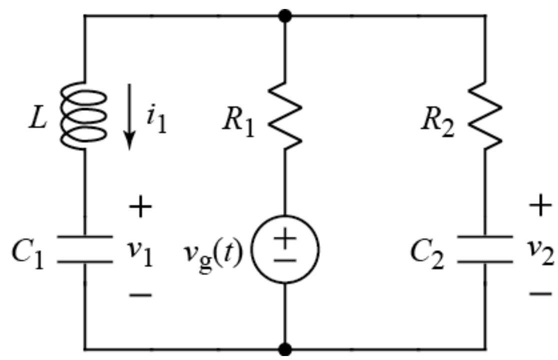
HOMEWORK #2 prob 3 solution

Ex:



At $t = 0$, $v_g(t)$ switches instantly from $-v_o$ to v_o.

a) Write the state-variable equations for the circuit in terms of the state vector:

$$\vec{x} = \begin{bmatrix} i_1 \\ v_1 \\ v_2 \end{bmatrix}$$

b) Evaluate the state vector at $t = 0^+$.

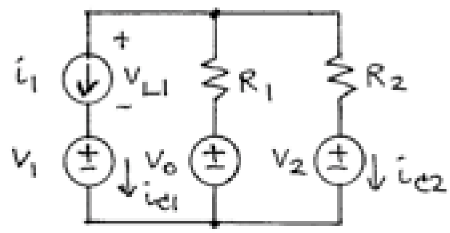
SOL'N: a) We put 1st derivatives of state vars on the left side. We then equate the 1st derivatives with non-derivatives via component eq'ns:

$$\frac{di_L}{dt} = \frac{v_L}{L} \quad \text{and} \quad \frac{dv_C}{dt} = \frac{i_C}{C}$$

Then we write v_L or i_C in terms of only state variables and components.

For the last step, it helps to replace L's with i sources and C's with v sources.

We also use $t > 0$, so $v_g = +v_o$.



$$\frac{di_1}{dt} = \frac{v_{L1}}{L} = \frac{1}{L}\left(v_2 + i_{c2} R_2\right)$$

we must use only i_1, v_1, v_2 so this must be replaced

$$\frac{dv_1}{dt} = \frac{i_{c1}}{C_1} = \frac{1}{C_1}\, i_1 \quad \text{(easy)}$$

$$\frac{dv_2}{dt} = \frac{i_{c2}}{C_2} = \frac{1}{C_2}\, i_{c2}$$

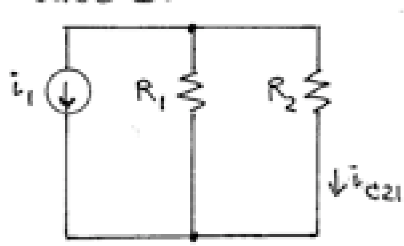
same as before; we must replace this with expression containing only $i_1, v_1,$ and v_2

Use superposition to find i_{c2}.

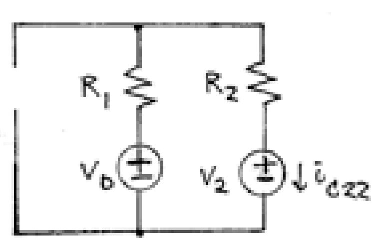
Before we start, we observe two things:

1) v_1 is in series with a current source and may be ignored, and
2) We can use just two cases — the first with only i_1 on, and the second with only v_0 and v_2 on.

case I:



case II:



For i_{c21}, we have an i-divider:

$$i_{c21} = -i_1 \frac{R_1}{R_1 + R_2}$$

For i_{c22}, we use the voltage across R_1 and R_2:

$$i_{c22} = \frac{v_0 - v_2}{R_1 + R_2}$$

So $$i_{c2} = -\frac{i_1 R_1}{R_1 + R_2} + \frac{v_0 - v_2}{R_1 + R_2}.$$

Our state-space eq'ns:

$$\frac{di_1}{dt} = \frac{1}{L}\left[v_2 - \frac{i_1 R_1 R_2}{R_1 + R_2} + \frac{(v_0 - v_2) R_2}{R_1 + R_2} \right]$$

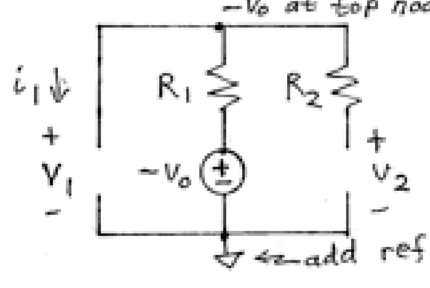
$$\frac{dv_1}{dt} = \frac{1}{C_1} i_1$$

$$\frac{dv_2}{dt} = \frac{1}{C_2}\left(-\frac{i_1 R_1}{R_1 + R_2} + \frac{v_0 - v_2}{R_1 + R_2} \right)$$

b) Because we are dealing with state variables, their values are the same at $t = 0^+$ as at $t = 0^-$.

At $t = 0^-$, L = wire, C_1 and C_2 = open.

$-v_0$ at top node since $i = 0$ (so no v-drop across R_1)



Note: $v_g = -v_0$ for $t < 0$.

No current flows, so the voltage across R_1 and R_2 is 0 V.

We have $-V_0$ at the top node, and we have $-V_0$ at the top of C_1 and C_2.

$$\text{So} \quad \underset{\displaystyle v_1(0^+)}{\overset{\|}{v_{c_1}(0^-)}} = \underset{\displaystyle v_2(0^+)}{\overset{\|}{v_{c_2}(0^-)}} = -V_0$$

We also have $i_1(0^-) = 0A$ because C_1 is open.

$$i_1(0^+) = 0 \text{ A}$$

$$v_1(0^+) = -V_0$$

$$v_2(0^+ = -V_0$$