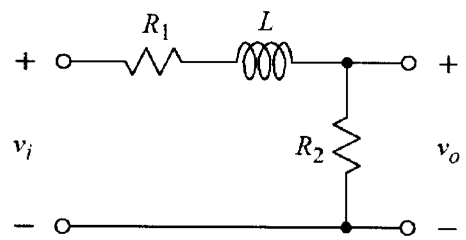
Ex:



$R_1 = 150\ \Omega$ $R_2 = 750\ \Omega$ $L = 1\ \mu\text{H}$

a) Determine the transfer function V_o/V_i. **Hint:** switch the order of R_1 and L and use a voltage divider.

b) Express the maximum of $|V_o / V_i|$ as a function of R_1 and R_2.

Sol'n: a) If we ignore the hint, we have the following calculation:

$$H(j\omega) = \frac{V_o}{V_i} = \frac{\cancel{V_i}}{\cancel{V_i}} \frac{R_2}{R_1 + R_2 + j\omega L}$$

Putting this in the form $H(j\omega) = k \dfrac{1}{1 \pm jx}$ is achieved by factoring out R_2 from the numerator and $R_1 + R_2$ from the denominator.

$$H(j\omega) = \frac{R_2}{R_1 + R_2} \cdot \frac{1}{1 + j\dfrac{\omega L}{R_1 + R_2}}$$

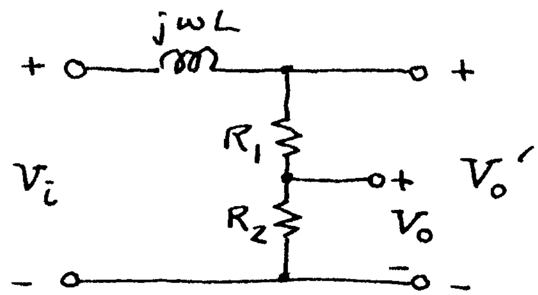
So $k = \dfrac{R_2}{R_1 + R_2}$ and $x = \dfrac{\omega L}{R_1 + R_2}$.

$$K = \frac{750\ \Omega}{150 + 750\ \Omega} = \frac{5}{6}$$

$$x = \frac{\omega\, 1\mu}{150 + 750\ \Omega} = \frac{\omega}{900\ \text{Mr/s}}$$

$$H(j\omega) = \frac{5}{6} \frac{1}{1 + j\dfrac{\omega}{900\ \mathrm{Mr/s}}}$$

If we use the hint, we view the filter as an RL filter, (with $R = R_1 + R_2$), and a V divider:



We have filter transfer function:

$$H'(j\omega) \equiv \frac{V_o'}{V_i} = \frac{R_1 + R_2}{R_1 + R_2 + j\omega L} = \frac{1}{1 + j\omega\dfrac{L}{R_1 + R_2}}$$

We multiply this by a voltage-divider gain term to our desired $H(j\omega)$:

$$V_o = V_o' \cdot \frac{R_2}{R_1 + R_2}$$

$$\text{or } \frac{V_o}{V_o'} = \frac{R_2}{R_1 + R_2}$$

$$\text{So } H(j\omega) = \frac{V_o}{V_i} = \frac{V_o'}{V_i} \cdot \frac{V_o}{V_o'}$$

$$\text{or } H(j\omega) = \frac{R_2}{R_1 + R_2} \cdot \frac{1}{1 + j\omega\dfrac{L}{R_1 + R_2}}$$

$$H(j\omega) = \frac{5}{6}\,\frac{1}{1 + j\,\dfrac{\omega}{900\,\mathrm{Mr/s}}}$$

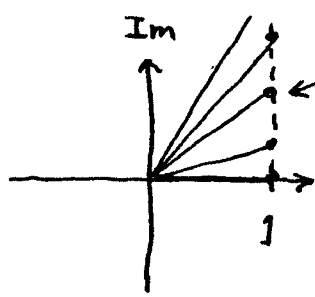
b) $$\max_{\omega}\left|\frac{V_o}{V_i}\right| \equiv \max_{\omega}\left|H(j\omega)\right|$$

$$= \frac{5}{6}\,\max_{\omega}\left|\frac{1}{1 + j\,\dfrac{\omega}{900\,\mathrm{Mr/s}}}\right|$$

$$= \frac{5}{6}\,\max_{\omega}\frac{1}{\left|1 + j\,\dfrac{\omega}{900\,\mathrm{Mr/s}}\right|}$$

$$= \frac{5}{6}\,\frac{1}{\min\limits_{\omega}\left|1 + j\,\underbrace{\dfrac{\omega}{900\,\mathrm{Mr/s}}}_{X}\right|}$$

We observe that X ranges from 0 to ∞ as ω ranges from 0 to ∞.



$1+jX$ is on this line. We want the shortest $1+jX$.

The shortest $1+jX$ is $1+j0$, with $|1+j0| = 1$.

So $$\max_{\omega}\left|\frac{V_o}{V_i}\right| = \frac{5}{6}\,\frac{1}{1} = \frac{5}{6} = \frac{R_2}{R_1+R_2}$$