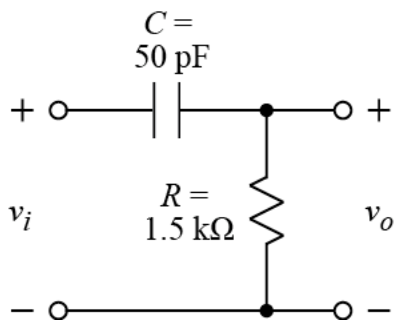
Ex:



$$v_i(t) = -4 + 2\sum_{k=1}^{\infty} \frac{1}{k^2+1} \sin(k\omega_0 t) \text{ V}$$

a) For the above circuit, determine the transfer function $H(j\omega) = V_o/V_i$.

b) Assume the circuit, has the following input signal:

$$v_i(t) = -4 + 2\sum_{k=1}^{\infty} \frac{1}{k^2+1} \sin(k\omega_0 t) \text{ V}$$

Note: $\omega_o = \frac{10}{3}$M r/s for the Fourier series.

Write the time-domain expression of the third harmonic (i.e., $k = 3$) of $v_o(t)$.

SOL'N: The third harmonic of $v_o(t)$, denoted here by $v_{o3}(t)$, is the response of the circuit to the third harmonic of the input signal,

$$v_{i3}(t) = 2 \cdot \frac{1}{k^2+1} \sin(k\omega_o t) \Big|_{k=3} \text{ V}.$$

or

$$v_{i3}(t) = \frac{2}{10} \sin(3\omega_o t) \text{ V} = \frac{1}{5} \sin\left(3 \cdot \frac{10}{3} \text{M} t\right) \text{V}$$

or

$$v_{i3}(t) = \frac{1}{5} \sin(10\text{M}t) \text{ V}$$

We convert $v_{i3}(t)$ to a phasor, V_{i3}, and compute impedance values for $\omega = 3\omega_o = 10$M r/s.

$$V_{i3} = -j\frac{1}{5}$$

$$\frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{-j}{10\text{M} \cdot 50\text{p}} \Omega = -j\, 2\text{k}\,\Omega$$

The output phasor, V_{o3}, is found using a voltage-divider formula.

$$V_{o3} = V_{i3} \cdot \frac{R}{R + \frac{1}{j\omega C}} = -j\,\frac{1}{5}\,\frac{1.5\cancel{K}}{1.5\cancel{K} - j2\cancel{K}}\ V$$

$$= -j\,\frac{1}{5}\,\frac{3}{3-j4}\ V = -j\,\frac{1}{5}\,\frac{3}{3-j4}\,\frac{3+j4}{3+j4}\ V$$

$$= -j\,\frac{1}{5}\,\frac{3(3+j4)}{25}\ V = \frac{1}{125}(12-j9)\ V$$

$$= \frac{8(12-j9)}{8(125)}\ V = 8(12-j9)\ mV$$

$$V_{o3} = 96 - j72\ mV$$

Thus, $v_{o3}(t) = 96mV\cos(10Mt) + 72mV\sin(10Mt)$

or $v_{o3}(t) = 120mV\cos(10Mt - 37°)$