Ex: Find the inverse Laplace transform for the following expression:

$$F(s) = \frac{5s+18}{s^2+6s}$$

SOL'N: We use partial fractions.

The denominator factors as $s(s+6)$.

$$F(s) = \frac{5s+18}{s^2+6s} = \frac{5s+18}{s(s+6)} = \frac{A}{s} + \frac{B}{s+6}$$

where $A = \left. \frac{F(s) \cdot s}{} \right|_{s=0} = \left. \frac{5s+18}{s+6} \right|_{s=0} = 3$

$$B = F(s)(s+6) \Big|_{s=-6} = \left. \frac{5s+18}{s} \right|_{s=-6} = 2$$

So $F(s) = \frac{3}{s} + \frac{2}{s+6}$

$$\mathcal{L}^{-1}\{F(s)\} = 3\,u(t) + 2e^{-6t}$$

We multiply the answer by $u(t)$ to remind ourselves that we cannot know $f(t<0)$ from a Laplace transform derived from an integral for $t>0^-$.

$$\mathcal{L}^{-1}\{F(s)\} = \left(3 + 2e^{-6t}\right)u(t)$$

Note: $u(t) \cdot u(t) = u(t)$