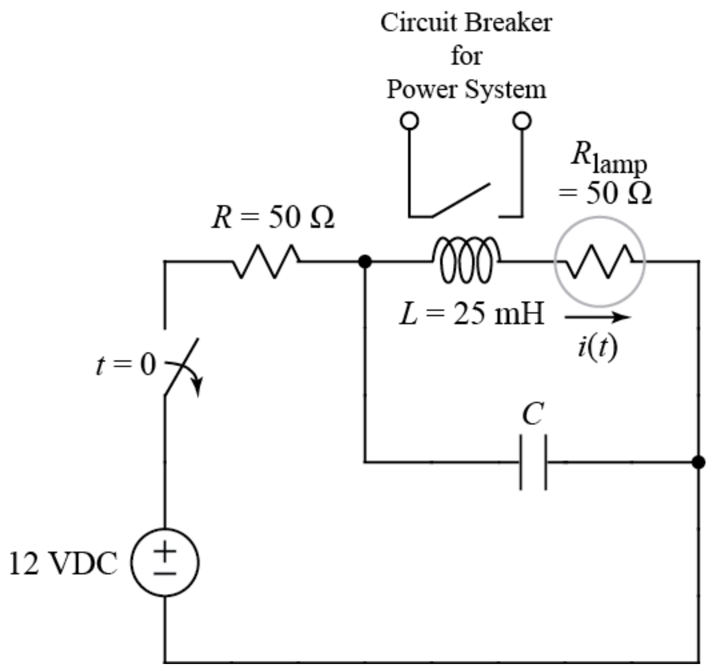
HOMEWORK #9 probs 1 and 2 solution

Ex:



After being closed for a long time, the switch opens at $t = 0$.

The above circuit shows an inductor acting as the electromagnet for a relay that holds a circuit breaker closed for a power system. A 6 V, 120 mA lamp, modeled as a 50 Ω resistor in the above diagram indicates whether the relay is on. When the switch is opened to turn off the relay, the capacitor in the circuit prevents a current spike from the inductor that would otherwise arc across the switch contacts. To turn the relay off as quickly as possible, the value of the capacitor is chosen to make the circuit critically damped.

a) Find the value of C that makes the circuit critically-damped after the switch is opened at $t = 0$.

b) Using the C value from problem 1, find a numerical expression for the lamp current, $i(t)$, for $t > 0$.

sol'n: a) When the switch is open, we have a series RLC circuit.

$$\alpha = \frac{R}{2L} \qquad \omega_0^2 = \frac{1}{LC}$$

The characteristic roots, as always, are given by the following quadratic equation:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Calculations:

$$\alpha = \frac{50\,\Omega}{2\,(25\,mH)} = 1k\ r/s$$

For critical damping, $\alpha = \omega_0$.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \alpha = 1k$$

$$\frac{1}{LC} = (1k)^2 = 1M$$

$$C = \frac{1}{1M \cdot L} = \frac{1\,\mu F}{25\,m} = 40\,\mu F$$

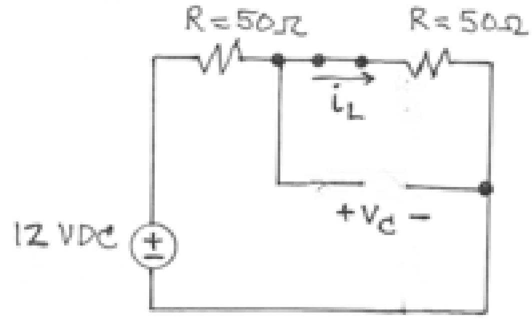
b) We use the solution form for critical damping:

$$i(t) = A_1 e^{-\alpha t} + A_2 t e^{-\alpha t} + A_3$$

A_3 will be zero since it equals the final value of $i(t)$ and there is no source of power in the circuit after $t=0$.

$$A_3 = 0\ A$$

Now we match the sol'n to initial conditions from the circuit. We begin by finding the state of the L and C at time $t=0^-$:



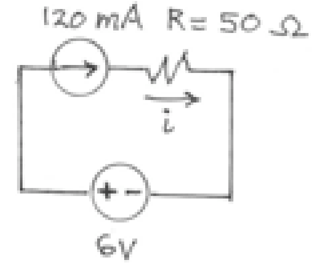
The L acts like a short and the C acts like an open. Thus, all current flows thru L.

$$i_L(0^-) = \frac{12\,V}{R+R} = \frac{12V}{100\Omega} = 120\,mA$$

$v_C(0^-)$ is the voltage across half the total resistance and so is half the total voltage:

$$v_C(0^-) = \frac{12V}{2} = 6\,V$$

At $t=0^+$, we model the L and C as sources.



Since the R is in series with a current source, it has the same current as the source.

$$i(0^+) = 120 \text{ mA}$$

We match this to our symbolic soln:

$$i(0^+) = A_1$$

Thus, $A_1 = i(0^+) = 120 \text{ mA}.$

Finally, we observe that $i = i_L$, so we can find the value of the derivativ of i by finding the derivative of i_L:

$$\left.\frac{di}{dt}\right|_{t=0^+} = \left.\frac{di_L}{dt}\right|_{t=0^+} = \frac{V_L(0^+)}{L}$$

Because R has 120 mA flowing thru it, it has v-drop $6V = 120mA \cdot 50\Omega$. The total v-drops around the loop must sum to zero. Thus, $V_L = 0V$.

$$\left.\frac{di}{dt}\right|_{t=0^+} = 0 \text{ A}$$

The symbolic soln gives

$$\left.\frac{di}{dt}\right|_{t=0^+} = -\alpha A_1 + A_2 = 0$$

Thus, $A_2 = \alpha A_1 = 1k \text{ r/s} \cdot 120 mA = 120 \text{ A/s}.$

$$i(t) = 120 mA\, e^{-1kt} + 120 A/s \cdot t\, e^{-1kt}$$