## LABORATORY PROJECT NO. 2 FREQUENCY RESPONSE AND FILTERS

## 1. Introduction

In many engineering applications, it is important to be able to select signals of a given frequency, or signals in a given band of frequencies. For example, the signal picked up by a radio antenna consists of signals generated by thousands of transmitting stations (plus noise signals) at frequencies that literally cover the spectrum. A device in the radio receiver called a tuner selects one station by allowing signals at that station frequency (actually a very narrow band of frequencies) to pass, while rejecting signals at other frequencies. Another example is multiplexing in communications systems. In this method, a unique band of frequencies is allocated to each information channel, and many channels are sent over one transmission medium. At the receiving end, frequency selection techniques are used to separate the channels. In touch-tone telephone systems, filters are used to differentiate the tones produced by the telephone buttons. Frequency-selective circuits are used in satellite communications, in television receivers, and in many other devices.

In this project, you will design a frequency-selective circuit based on the circuit shown in Fig. 1. This circuit will strongly reject one frequency and strongly pass another frequency, with intermediate response for other frequencies. More specifically, you will design this circuit to reject a specific first harmonic and pass a specific third harmonic. This circuit is thus a combination of a bandpass and a bandreject filter.

Be sure that you measure the inductance, shunt capacitance, and dc resistance of the inductor during the first lab period, as you will need these values for your calculations.


Fig 1. (a) A circuit designed to reject a specific fundamental and pass a specific third harmonic. The combination of $v_{s}(t)$ and $R_{g}$ is a model of a generator, with $v_{g}(t)$ representing its terminal voltage. The inductor is modeled by a simple inductance. (b) A more sophisticated model of an inductor than a simple inductance; to be used in a later part of the design.

## 2. Measurement of Circuit Components

A. Inductance. Use either of the inductors mounted on the lab-bench console or check out a similar one from the stockroom. Measure the inductance of the inductor using a suitable meter.
B. Inductor Resistance. Measure the dc resistance of the inductor. This dc resistance will have the same order of magnitude as $R_{s}$ in the inductor model, but will not be equal to $R_{s}$ because $R_{s}$ changes with frequency. Since measuring $R_{s}$ as a function of frequency is complicated and requires more time than you have to spend on it, use the dc resistance of the inductor as a rough approximation to $\mathrm{R}_{\mathrm{s}}$ in the work that follows.
C. Distributed Capacitance of the Inductor. Measure the distributed capacitance of the inductor. This corresponds to $\mathrm{C}_{\mathrm{s}}$ in Fig. 1(b), in which the inductor is modeled by the $\mathrm{R}_{\mathrm{s}}, \mathrm{L}$, and $\mathrm{C}_{\mathrm{S}}$ combination as shown. One way to measure $\mathrm{C}_{\mathrm{s}}$ is to connect a large resistor in series with the inductor and a sinusoidal generator and adjust the frequency until parallel resonance of $L$ and $C_{S}$ occurs. Then $C_{S}$ can be calculated using the measured values of the resonant frequency and the inductance $\left(\mathrm{R}_{\mathrm{s}}\right.$ can usually be neglected). The frequency at which the parallel LC resonance occurs is the frequency at which the voltage across the large series resistor is minimum.

## 3. Preliminary Calculations

The design objective is a circuit that will reject a 1 kHz fundamental and pass a 3 kHz third harmonic. The preliminary calculations of this section are to help you understand how the circuit works in preparation for designing it.
A. Sum a Fourier Series. Write a MATLAB program that sums the Fourier series terms for an arbitrary periodic function of time and plots the results for at least one period. You will use this program (or modifications of it) several times in the following parts of this project. (You may wish to refer to the MATLAB demonstration program on summing a Fourier series.) Using your program, sum the Fourier series for a triangular wave and plot it. Include enough terms in the sum to get a reasonably good triangular wave.

## B. Calculate the Output Voltage.

1. Write $\mathrm{v}_{\mathrm{g}}(\mathrm{t})$ in Fig. 1 as a Fourier series with arbitrary $\mathrm{a}_{\mathrm{n}}$ and $\mathrm{b}_{\mathrm{n}}$, use superposition in the time domain, transform the circuit in Fig. 1 for each harmonic to the frequency domain, solve for the phasor voltage $\mathbf{V}_{\text {on }}$ for each harmonic, and transform back to the time domain to get the Fourierseries representation of $v_{o}(t)$ in terms of $a_{n}, b_{n}$ and the circuit components. Don't do any more algebraic manipulations on the expressions than you need to. For example, in deriving the expression for $\mathbf{V}_{\text {on }}$, you will need an expression for the complex impedance $\mathrm{Z}_{\mathrm{ab}}$. Instead of reducing the expression for $\mathrm{Z}_{\mathrm{ab}}$ algebraically, leave it to MATLAB to calculate if from $1 / \mathrm{jn} \omega_{\mathrm{o}} \mathrm{C}_{1}$, etc. On the basis of the MATLAB program you wrote in 3A above, write a second MATLAB program to calculate and plot $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$ for any set of circuit parameters.
2. Plot $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$ for the following circuit parameters:
$v_{g}(t)$ is a triangular wave with a period of 1 ms

$$
\begin{array}{ll}
\mathrm{R}_{1}=10 \mathrm{k} \Omega & \mathrm{~L}=100 \mathrm{mH} \\
\mathrm{C}_{1}=22 \mathrm{nF} & \mathrm{R}_{\mathrm{S}}=1 \Omega \\
\mathrm{C}_{2}=176 \mathrm{nF} & \mathrm{C}_{\mathrm{S}}=185 \mathrm{pF}
\end{array}
$$

3. Determine the frequency response of the circuit. The frequency response is obtained by letting $\mathrm{v}_{\mathrm{S}}(\mathrm{t})$ be a sinusoidal voltage source of radian frequency $\omega$ and then calculating or measuring $\left|\mathbf{V}_{\mathrm{o}} / \mathbf{V}_{\mathrm{g}}\right|$, the magnitude of the transfer function, as a function of $\omega$ or f , where $\omega=2 \mathrm{ff}$ ( f will probably be more convenient). Calculate the frequency response as follows:
a. Write a MATLAB program that will calculate $\left|\mathbf{V}_{\mathrm{o}} / \mathbf{V}_{\mathrm{g}}\right|$ and plot it versus f for any set of circuit parameters.
b. Using this MATLAB program, plot the frequency response $\left(\left|\mathbf{V}_{\mathrm{o}} / \mathbf{V}_{\mathrm{g}}\right|\right)$ for the same set of circuit parameters you used in Part 3B2 above
4. On the basis of the frequency response you plotted and the harmonics of the triangular waveform $\mathrm{vg}_{\mathrm{g}}(\mathrm{t})$, explain the shape of the $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$ you plotted in Part 3B2 above.
5. Plot $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$ as in Part 3B2 above but for the period of $\mathrm{vg}_{\mathrm{g}}(\mathrm{t})$ equal to 0.6 ms instead of 1 ms . On the basis of the frequency response and the harmonics of $\mathrm{v}_{\mathrm{g}}(\mathrm{t})$, explain the shape of this $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$. Now let the period of $\mathrm{vg}_{\mathrm{g}}(\mathrm{t})$ be 0.834 ms , plot $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$ and explain its shape on the basis of the frequency response.

## 4. Circuit Design

A. Approximate Equations for Frequency Response. Neglecting $\mathrm{R}_{\mathrm{s}}$ and $\mathrm{C}_{\mathrm{s}}$, transform the circuit of Fig. 1 to the frequency domain and write an expression for $\mathbf{V}_{\mathrm{o}} / \mathbf{V}_{\mathrm{g}}$. Since $\mathrm{R}_{\mathrm{s}}$ and $\mathrm{C}_{\mathrm{s}}$ are neglected, this expression is approximate. Make consistency checks on the expression.
B. Using these approximate equations, choose component values that will reject a fundamental at $1 \mathrm{kHz}\left(\mathbf{V}_{\mathrm{O}}=0\right)$ and pass a third harmonic at $3 \mathrm{kHz}\left(\mathbf{V}_{\mathrm{O}}=\max \right)$. Using the MATLAB program you wrote, plot $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$ vs t and $\left|\mathbf{V}_{\mathrm{o}} / \mathbf{V}_{\mathrm{g}}\right|$ for these component values (use
your measured values of $L, R_{s}$, and $C_{S}$ ) when $\mathrm{vg}_{\mathrm{g}}(\mathrm{t})$ is a triangular wave with a fundamental of 1 kHz . Explain the shape of $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$ on the basis of the frequency-response plot. Note that even though the fundamental is suppressed and the third harmonic is maximized, the higher harmonics will all be present in $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$.
C. Using these approximate equations, choose component values that will reject a fundamental at 9 kHz and pass a third harmonic at 27 kHz . Plot $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$ vs t and $\left|\mathbf{V}_{\mathrm{o}} / \mathbf{V}_{\mathrm{g}}\right|$ for these component values (use your measured values of $\mathrm{L}, \mathrm{R}_{\mathrm{S}}$, and $\mathrm{C}_{\mathrm{s}}$ ). Explain why the approximate design equations don't give good results for rejecting a fundamental at 9 kHz , and why $\mathrm{R}_{\mathrm{S}}$ and $\mathrm{C}_{\mathrm{S}}$ affect the circuit response more at the higher frequencies than at the lower frequencies.

## 5. Measurements

Find circuit components with values as close as you can to those you calculated in 4B above to reject a fundamental near 1 kHz and pass the corresponding third harmonic. You may vary the frequency of the fundamental slightly to get the best fundamental suppression and third harmonic enhancement with available capacitors. Measure the values of all the circuit components you select.
A. Frequency Response. With these measured circuit components, measure the frequency response by measuring $\left|\mathbf{V}_{\mathrm{o}}(\mathrm{f}) / \mathbf{V}_{\mathrm{g}}(\mathrm{f})\right|$ with a sinusoidal generator voltage. Take the data carefully so you can compare it with calculated data. Be sure you have measured values of all components to use in your calculations. Be sure to get values for $f$ corresponding to the fundamental and third harmonic.
B. Time-Domain Reponse. Change the generator voltage to a triangular wave with a fundamental frequency of 1 kHz and display the output voltage on an oscilloscope. Vary the frequency slightly to get the best third harmonic on the output. Measure the frequency of the fundamental. Record the waveform of $v_{0}(t)$ for comparison with calculated results.

## 6. Comparisons of Calculated and Measured Values

A. Calculations with Measured Component Values. Using your MATLAB programs, plot $\left|\mathbf{V}_{0} / \mathbf{V}_{\mathrm{g}}\right|$ vs f and $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$ vs t for the measured values of circuit components, and with the measured value of the triangular-wave period that gave the best third-harmonic output.
B. Sinusoidal Frequency Response. Compare your calculated and measured values of $\left|\mathbf{V}_{\mathrm{o}} / \mathbf{V}_{\mathrm{g}}\right|$ vs f by plotting them both on the same set of axes, clearly labeling each.
C. Triangular-Wave Response. Compare calculated and measured values of $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$ by plotting them on the same set of axes, clearly labeling each.

## 7. Formal Report

Write a formal report describing your work on this project. See instructions in "Course Procedures" about how to write the report. Include at least the following in your report.

1. A short introduction. (You may also attach this handout to the report and refer to it so that you don't have to copy the information in it.)
2. A careful description of all the work that you did.
a. Give clear derivations of mathematical expressions. Include consistency checks. Include listings of your MATLAB programs in an appendix, with explanation (may be in the form of comment statements in the program).
b. Explain all measurements carefully and include data appropriately in clearly labeled tables.
c. Give a clear comparison of calculated and measured values, plotting them on the same set of axes. Explain why the calculated and measured values are not the same.
3. Conclusions, including:
a. A discussion of the validity of the models used.
b. A discussion of the success of your design procedure and the usefulness of the circuit for rejecting some frequencies and passing others.

## 8. Your Grade

Your report will be graded according to the following:

## Category

Percentage

| * Communication | 30 |
| :--- | :--- |
| Technical Content: |  |

2. Component Measurements 5
3. Preliminary Work 20
4. Circuit Design 20
5. Measurements 15
6. Comparison of Calculated and Measured Results5
7. Conclusions ..... 5
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[^0]:    * According to the items listed on p. 4 in "Course Procedure."

