# LABORATORY PROJECT NO. 3
# THE SPIROGRAVITATOR

Captain's Log
Stardate 4.2576

This is Captain Quirk of the Starship Surprise. We are surrounded by Klingulon ships in quadrant 19, sector 80. The Klingulons are closing and preparing to attack. We are outnumbered 35 to 1 and have no operational photon torpedoes or phasor banks. The situation is critical. We have no choice but to use the Federation's new weapon, the spirogravitator, untried as it may be. Mr. Shock informs me the new weapon may only be used in spiral form; at this time it is impossible to use it as a normal straight-line weapon due to the untold possibilities and problems which may exist. Mr. Shock has already plotted the Klingulon vessels, but it is impossible for him to plot the spiral path of the weapon. All Vulcans have a mind block against spirals and cannot function in any way if a spiral is visible in the immediate surroundings. However, it is imperative that the spiral be plotted on the scope before the spirogravitator is used, to be sure that none of the nearby planets are in the weapon's path. We are now screening the crew members to find someone with the spiral-plotting ability.

The circuit diagram of the spirogravitator is shown in Fig. 1. $v_i$ is a rectangular wave. The voltage $v_0$ is connected to the horizontal input of an oscilloscope and $v_1$ is connected to the horizontal input of the same oscilloscope. The design problem is to select circuit components so that $v_0$ and $v_1$ will have equal amplitudes, and will be underdamped with the same frequency of oscillation and the same damping rate, but 90° out of phase. With these conditions, the resulting pattern on the oscilloscope will be the desired double spiral.

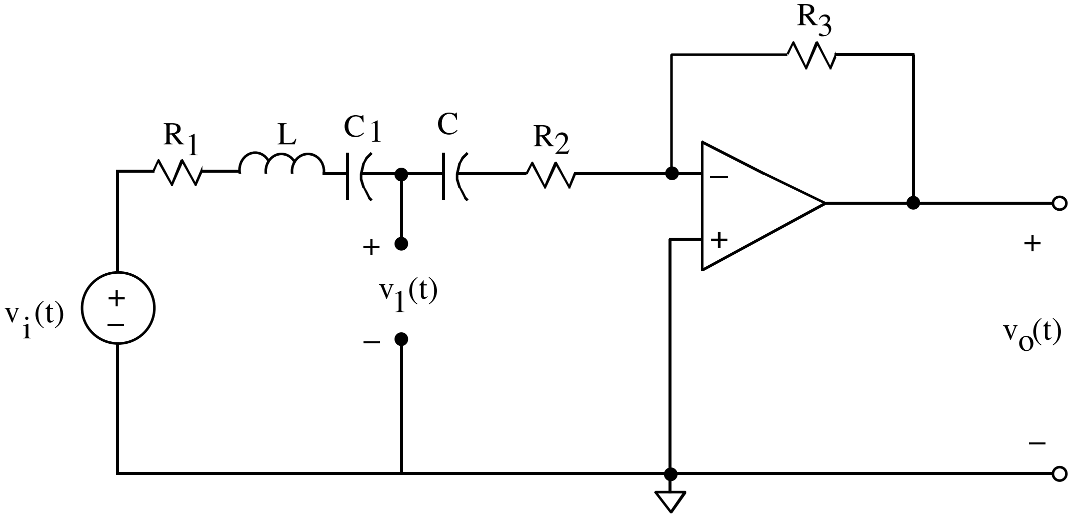

Fig. 2.1. Diagram of the spiralgraph circuit. $v_i$ is a rectangular wave with zero dc level.

## 1. Model of the Device

For the inductance in the circuit of Fig. 2.1, use either the 100 mH or the 200 mH inductor on the lab-bench consoles. Measure both the inductance and dc resistance of the inductor you use. Do this during the first lab period, as you will need these values for the calculations that you will be making. If you make $R_1$ and $R_2$ large compared to $R_0$, the resistance of the inductor, you may be able to neglect $R_0$. You will probably find that the shunt capacitance of the inductor and the shunt capacitance of the inputs to the oscilloscope are both negligible here, but that is not always the case.

## 2. Circuit Design

**2.1. Equations.** Transform the circuit of Fig. 1 to the s domain and write expressions for $V_1(s)$ and $V_0(s)$, first for a half-period interval when $v_i(t)$ is positive, and then for a half-period when $v_i(t)$ is negative. Include initial conditions for the capacitance and inductance. Make consistency checks to ensure that your expressions are accurate. Take the inverse transform of $V_1(s)$ and $V_0(s)$. One of them will be of the form

$$a\,e^{-\alpha t}\sin\beta t$$

and the other of the form

$$b\,e^{-\alpha t}\sin(\beta t+\psi)+c.$$

**2.2. Circuit parameters.** Choose circuit parameters so that

$$\psi = \pm 90°$$

$$a = b$$

and so that

$$\frac{1}{\alpha} \geq T$$

where $\beta = 2\pi/T$ and T is the period of oscillation. The first requirement is to ensure that $v_1(t)$ and $v_0(t)$ are damped sinusoids that differ in phase by 90°. The second makes the amplitudes equal. The third is to ensure that the waves do not damp out too fast. Be sure that the total series resistance is large enough that the generator is not loaded excessively (the current is not too high) because excessive loading will distort the generator waveform.

**2.3. Double spiral.** Use MATLAB to plot the spirals that you expect on the oscilloscope by letting $x = v_0(t)$ and $y = v_1(t)$ and plotting y versus x. Do this for at least one full period of oscillation both when $v_i(t)$ is positive and when it is negative. Be sure you understand fully how the spirals are produced on the screen when $v_0(t)$ and $v_1(t)$ are connected to the horizontal and vertical inputs, respectively.

## 3. Measurements

**3.1. Construction.** Construct a breadboard circuit using the component values that you have selected.

**3.2. Display $v_0(t)$ and $v_1(t)$.** Display $v_i$ and $v_0$ simultaneously on a dual-trace oscilloscope and be sure that $v_i$ is a good rectangular wave and that the period of $v_i$ is enough longer than the decay time of $v_0$ that the natural response of $v_0$ is negligible after half a period of $v_i$.

Display $v_0$ and $v_1$ simultaneously on a dual-trace oscilloscope and check to see that they are consistent with your calculations, but don't make detailed measurements until you get the spiralgraph to work. Be sure that $v_0$ and $v_1$ are 90° out of phase, and their amplitudes are equal.

**3.3. Display the spirals.** Connect $v_0$ and $v_1$ appropriately to the x-y inputs of an oscilloscope. Adjust the amplitude of $v_i$ until a good double spiral is obtained. Set the generator frequency high enough that the dots at the center of the spirals are not too bright.

**3.4. Measure $v_0(t)$ and $v_1(t)$.** Display $v_0$ and $v_1$ again with the dual trace and measure $\alpha$, $\beta$, a, b, c, and $\psi$ to compare with your calculations. Also measure the actual values of your components and use the measured values in your calculations to get the best correspondence with measurements.

**3.5. Measure the spirals.** Display the spirals again and measure the parameters of the pattern so that you can compare the calculated and measured patterns.

## 4. Comparison of Calculated and Measured Results

**4.1. $v_0(t)$ and $v_1(t)$.** Compare the calculated and measured values of $v_0(t)$ and $v_1(t)$ by plotting measured and calculated values of $v_0(t)$ on the same set of axes and by plotting the measured and calculated values of $v_1(t)$ on the same set of axes.

**4.2. Spirals.** Compare the calculated and measured values of the spiral patterns by plotting them on the same set of axes.

## 5. Formal Report

Write a formal report describing your work on this project. See instructions in "Course Procedures" about how to write it. Include at least the following:

1. A short introduction. You may attach this handout to the report and refer to it so that you don't have to copy the information it contains.
2. A careful description of the work that you did in Sections 1-4 above.
   a. Give clear derivations of mathematical expressions. Include consistency checks.
   b. Give a careful, clear explanation of how and why you selected values of circuit components.

c. Give a clear explanation of how $v_0$ and $v_1$ produce a double spiral.

d. Explain all measurements carefully and include data in clearly labeled tables.

e. Compare calculated and measured values of $v_0(t)$ and $v_1(t)$ showing both measured and calculated values plotted on a common set of axes.

f. Compare the measured spirals and the calculated spirals.

3. Conclusions, including a discussion of the validity of models used in calculations and an explanation of differences between calculated and measured values.

## 6. Your Grade

Your report will be graded according to the following:

| Category | Percentage |
|---|---|
| [*] Communication | 30 |
| Technical Content: | |
| 1. Component Measurements | 10 |
| 2. Circuit Design | 20 |
| 3. Measurements | 20 |
| 4. Comparison | 15 |
| 5. Conclusions | 5 |
| | 100 |

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[*] According to the items listed in p. 4 in "Course Procedures."