4. (50 points)

\[ Z_1 = (5 - j5) \Omega \quad Z_2 = (20 + j20) \Omega \]

a. Find the input impedance, \( z_{in} = \frac{V_1}{I_1} \), for the above circuit.

b. Using \( z_{in} \) from (a), find a numerical expression for \( V_{AB} \) in the circuit below.

Balanced three-phase system.

\[ V_{an} = 52 \angle 0^\circ \text{V} \quad V_{bn} = 52 \angle -120^\circ \text{A} \quad z_{line} = j12 \Omega \]

ans: a) \( z_{in} = 5 \Omega \)

b) \( V_{AB} \approx 234 \angle -37.38^\circ \text{V} \)

sol'n: (a) Transformer is ideal. To distinguish currents in the transformer itself from other currents, we use a prime to denote the transformer currents. The current flowing into the dot on the primary side is \( I'_1 \), and the current flowing out of the dot on the secondary side is \( I'_2 \):

\[ \frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \frac{I'_1}{I'_2} = \frac{N_2}{N_1} \]
Using the above model, we can derive the formula (or we can just look up the formula) for secondary impedance reflected into the primary:

\[ z_r = \left( \frac{N_1}{N_2} \right)^2 z_2 \]

Our model, given \( N_1/N_2 = 1/2 \) turns ratio, is:

\[ z = z_1 \parallel z_r = (5 - j5) \parallel (5 + j5) \Omega \]

\[ z = \frac{(5 - j5)(5 + j5)}{5 - j5 + 5 + j5} = \frac{5^2 + 5^2}{10} = 5 \Omega \]

sol'n: (b) Our first step is to convert our circuit to a Y – Y form so we can use a single-phase equivalent model. In this problem, the circuit is already in Y – Y form and we may draw the single-phase equivalent directly:

We find \( V_{AN} \) and then calculate \( V_{AB} \) using phasor diagrams. We obtain \( V_{AN} \) from the voltage divider formula:

\[ V_{AN} = V_{a'n} \frac{z_{in} - j12\Omega}{z_{line} + z_{in} - j12\Omega} \]

\[ V_{AN} = 52 \angle 0^\circ V \frac{5\Omega - j12\Omega}{j12\Omega + 5\Omega - j12\Omega} = 52 \angle 0^\circ V \frac{5\Omega - j12\Omega}{5\Omega} \]

\[ V_{AN} = 52 \angle 0^\circ V \frac{13\Omega - 67.38^\circ \Omega}{5\Omega} \]

We use a phasor diagram to relate \( V_{AN} \) to \( V_{AB} \). The diagram shows the relationship between \( V_{AN} \) and \( V_{AB} \), and we assume \( V_{AN} \) has phase angle zero so we can find the relative phase angle of \( V_{AB} \).
From the diagram, we deduce that

\[ V_{AB} = V_{AN} \cdot \sqrt{3} \angle 30^\circ \]

Plugging in the value of \( V_{AN} \) gives the numerical value of \( V_{AB} \).

\[ V_{AB} = 52 \angle 0^\circ V \cdot \frac{13 \angle -67.38^\circ \Omega}{5 \Omega} \cdot \sqrt{3} \angle 30^\circ = \frac{52 \cdot 13 \cdot \sqrt{3}}{5} \angle -37.38^\circ V \]

\[ V_{AB} \approx 234 \angle -37.38^\circ \text{ V} \]