1. (25 points)

\[ \begin{align*}
    v_g & \text{ is a dc voltage source} \\
    & \text{After having been open for a long time, the switch is closed at } t = 0. \\
    
    \text{a. Give expressions for } i_1(0+) \text{ and } i_1'(0+), \text{ (i.e., } di_1/dt \text{ at } t = 0^+), \text{ in terms of no} \\
    & \text{more than } v_g, R_0, R, L, \text{ and } C. \\
    
    \text{b. For } L = 10 \mu \text{H}, \text{ choose } R \text{ and } C \text{ so that the system is underdamped and} \\
    & \alpha = 3 \cdot 10^6 \text{ rad/s, } \omega_d = 4 \cdot 10^6 \text{ rad/s.} \\
\end{align*} \]

\text{ans: a) } i_1(0^+) = 0 \text{A} \\
\frac{di_1(t)}{dt} \bigg|_{t=0^+} = -\frac{v_g}{L} \frac{R}{R + R_0} \\

\text{b) } C = 4 \text{nF, } R = 41.7 \Omega \\

\text{sol'n: (a) To determine what the } L \text{ and } C \text{ are doing at } t = 0^+, \text{ we find the value of} \\
\text{ } i_L(t = 0^-) \text{ and } v_C(t = 0^-) \text{ and argue that these values will be the same at time} \\
\text{ } t = 0^+, \text{ since they cannot change instantly.} \\
\text{We find } i_L(t = 0^-) \text{ and } v_C(t = 0^-) \text{ from circuit model for } t = 0^-: \text{ } L \text{ acts like} \\
\text{wire, } C \text{ acts like open.} \\

\[ \text{(C open circuit)} \]
Current circulates only in the left inner loop, meaning \( i_L = -i_R \). The current in this loop is just the source voltage divided by the sum of \( R \) and \( R_o \).

\[
i_L(t = 0^-) = -\frac{v_g}{R + R_o}
\]

The right side looks like two dangling wires carrying no current but measuring the voltage drop, \( v_C \), across \( R \). The voltage drop is given by the voltage divider formula for \( R \) and \( R_o \) across \( v_g \).

\[
v_C(t = 0^-) = v_g \frac{R}{R + R_o} \quad \text{circuit is} \quad v_g
\]

Since \( i_L \) and \( v_C \) cannot change instantly, we have

\[
i_L(t = 0^+) = i_L(t = 0^-) = -\frac{v_g}{R + R_o},
\]

\[
v_C(t = 0^+) = v_C(t = 0^-) = v_g \frac{R}{R + R_o}.
\]

The instant after the switch is closed, we model the inductor as a current source and the capacitor as a voltage source:

Henceforth, we work towards final answers involving only the state variables \( i_L \) and \( v_C \). Since these are the only sources in the circuit, our answers will be in terms of state variables if we solve the circuit using standard techniques such as node-voltage.

If we place a reference on the bottom center node, then the top center node voltage is \( v_C(0^+) \), owing to the voltage source on the right. Since \( i_1 \) is a current flowing through a voltage source, we find its value by summing currents out of the top-center node.

\[
i_L + i_R + i_1 = 0A
\]

\[
i_1(0^+) = -i_L(0^+) - \frac{v_C(0^+)}{R} = \frac{v_g}{R + R_o} - \frac{v_g}{R + R_o} = 0A
\]
This answer seems suspicious because the current before \( t = 0^+ \) was zero, too, and something has to change... The answer is correct, however, because the inductor and resistor currents have not changed. They are both determined by state variables that have not changed.

Qualitatively, what happens in this circuit is that \( i_L \) cannot change instantly when the switch closes. Before the switch closes, \( R \) carries all of \( i_L \) (because \( C \) acts like open circuit). After the switch closes, \( i_R \) cannot change instantly (because \( v_C \) is across \( R \) and \( v_C \) cannot change instantly). Thus, \( R \) still carries \( i_L \) just after the switch closes. This implies that \( i_1(0^+) = 0 \).

To find the derivative of \( i_1 \) at \( t = 0^+ \), we differentiate the equation for \( i_1 \) in terms of state variables found above and plug in \( t = 0^+ \) after we differentiate. (Note that, if we differentiate \( i_L(0^+) \), then we are differentiating a constant and we get zero.)

\[
i_1(t) = -i_L(t) - \frac{v_C(t)}{R}
\]

\[
\frac{di_1(t)}{dt} = - \frac{di_L(t)}{dt} - \frac{1}{R} \frac{dv_C(t)}{dt}
\]

We convert these derivatives of state variables back into variables that are not derivatives by rearranging basic component equations for \( L \) and \( C \):

\[
v_L(t) = L \frac{di_L(t)}{dt} \Rightarrow \frac{di_L(t)}{dt} = \frac{v_L(t)}{L}
\]

\[
i_C(t) = C \frac{dv_C(t)}{dt} \Rightarrow \frac{dv_C(t)}{dt} = \frac{i_C(t)}{C}
\]

We have

\[
\frac{di_1(t)}{dt} \bigg|_{t=0^+} = - \frac{di_L(t)}{dt} \bigg|_{t=0^+} - \frac{1}{R} \frac{dv_C(t)}{dt} \bigg|_{t=0^+}
\]

\[
\frac{di_1(t)}{dt} \bigg|_{t=0^+} = - \frac{v_L(t)}{L} \bigg|_{t=0^+} - \frac{1}{R} \frac{i_C(t)}{C} \bigg|_{t=0^+}
\]

\[
\frac{di_1(t)}{dt} \bigg|_{t=0^+} = - \frac{v_L(0^+)}{L} - \frac{1}{R} \frac{i_C(0^+)}{C}
\]

Returning to the circuit diagram for \( t = 0^+ \), we solve for \( v_L(0^+) \) and \( i_C(0^+) \) using standard methods. In this case, we have \( v_L(0^+) = v_C(0^+) \) and \( i_C(0^+) = i_1(0^+) \). Plugging in these known values, we have
\[
\frac{di_1(t)}{dt} \bigg|_{t=0^+} = -\frac{v_C(0^+)}{L} - \frac{1}{R} i_1(0^+) = -\frac{v_g}{L} \frac{R}{R + R_o} - \frac{1}{2} 0A
\]

\[
\frac{di_1(t)}{dt} \bigg|_{t=0^+} = -\frac{v_g}{L} \frac{R}{R + R_o}.
\]

**sol'n:** (b) \( \alpha = 3 \text{ M/s} = 1/(2RC) \) for parallel RLC

\[
\omega_d = 4 \text{ M/s} = \sqrt{\omega_o^2 - \alpha^2} \quad \text{underdamped RLC}
\]

Thus, \( \omega_d^2 = 4^2 \text{ M}^2/\text{s}^2 = \omega_o^2 - 3^2 \text{ M}^2/\text{s}^2 \).

From \( 3^2 + 4^2 = 5^2 \) Pythagorean triple we have

\[
\omega_o^2 = 5^2 \text{ M}^2/\text{s}^2
\]

\[
\omega_o^2 = \frac{1}{LC} \Rightarrow C = \frac{1}{\omega_o L} = \frac{1}{5^2 \text{ M}^2/10 \mu} = \frac{1}{250 \text{ M}} \text{ F}
\]

\[
C = \frac{1}{250} \mu \text{ F} = \frac{1}{250} \text{ nF} = 4 \text{ nF}.
\]

Now use the definition of alpha

\[
\alpha = \frac{1}{2RC} \Rightarrow R = \frac{1}{2\alpha C} = \frac{1}{2(3) \text{ M}} \frac{\Omega}{4 \text{n}} = \frac{1}{24} 41.7 \Omega.
\]