3. (50 points)

![Circuit Diagram]

\[ I_A = 1 \text{ A} \]
\[ R = 2400 \Omega \]
\[ L = 200 \mu \text{H} \]
\[ C = 50 \text{ pF} \]

a. After being closed for a long time, the switch is opened at \( t = 0 \). Write a numerical time-domain expression for \( i(t) \), the current through the capacitance. This expression must not contain any complex numbers.

b. State whether \( i(t) \) is underdamped, overdamped, or critically damped.

**ans:** a) \( i(t > 0) = \left( -\frac{1}{2} \cos 8Mt - \frac{3}{8} \sin 8Mt \right) e^{-6Mt} \text{ A} \)

b) Underdamped.

**sol'n:** (a) When the switch is open, we have series RLC.

\[ \alpha = \frac{R}{2L} = \frac{2.4k}{2.200\mu} = \frac{1.4k}{400} \quad \text{M} = \frac{6}{s} \]
\[ \omega_o^2 = \frac{1}{LC} = \frac{1}{200\mu \cdot 50p} = \frac{1}{10k \cdot 10k} = \frac{100 \text{ M}^2}{s^2} \]

\[ \therefore \quad \omega_o = 10 \text{ M} / s \]
\[ \omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{(10 \text{ M})^2 - (6 \text{ M})^2} = 8 \text{ M} / s \quad \left( 6^2 + 8^2 = 10^2 \right) \]

Now find initial condition (i.e. \( i \) and \( di/dt \), or \( v \) and \( dv/dt \) at \( t = 0^+ \)).

\[ i_L(t = 0^+) = i_L(t = 0^-) \]
\[ v_C(t = 0^+) = v_C(t = 0^-) \]

\[ \left\{ \begin{array}{l}
\text{cannot change instantly}
\end{array} \right. \]
Circuit for t = 0: L's = wires, C's = open circuits.

\[ i_L(t=0^-) = \frac{1}{2} \text{A} \quad \text{(current divider)} \]

\[ v_C(t = 0^-) = I_A R \parallel R = I_A \cdot \frac{R}{2} = 1 \text{A} \cdot 1.2 \text{kΩ} = 1.2 \text{kV} \]

\[ \therefore \quad i_L(t = 0^+) = \frac{1}{2} \text{A}, \quad v_C(t = 0^+) = 1.2 \text{kV} \]

After the switch is open, \( i = -i_L \) since C and L are in series.

\[ \therefore \text{ Solve for } i_L, \text{ and then change the sign. Note that } i_L \text{ is the variable in the differential equation for a series RLC. Thus, we know how to find it.} \]

We also need

\[ \left. \frac{di_L(t)}{dt} \right|_{t=0^+}. \]

Use V-loop for RLC at \( t = 0^+ \):

\[ L \frac{di_L}{dt} + i_L R - v_C = 0 \text{ V.} \]

Note that at \( t = 0^+ \), \( i_R = i_L \) since R, L in series,

\[ \left. \frac{di_L}{dt} \right|_{t=0^+} = \frac{-i_L(t = 0^+)R + v_C(t = 0^+)}{L} \]

\[ \left. \frac{di_L}{dt} \right|_{t=0^+} = \frac{-\frac{1}{2} \text{A} \cdot 2.4 \text{kΩ} + 1.2 \text{kΩ}}{200 \mu\text{H}} \]

\[ \left. \frac{di_L}{dt} \right|_{t=0^+} = 0 \text{ A/s} \]

Now use general underdamped solution:

\[ i_L(t) = (B_1 \cos \omega_d t + B_2 \sin \omega_d t)e^{-\alpha t} \]
\[ B_1 = i_L(t = 0^+), \quad -\alpha B_1 + \omega_d B_2 = \frac{di_L}{dt} \bigg|_{t=0^+} = 0 \text{ A/s} \]

\[ \therefore B_1 = \frac{1}{2} \text{ A}, \quad B_2 = \frac{\alpha B_1}{\omega_d} = \frac{6M}{8M} \frac{1}{2} \text{ A} = \frac{3}{8} \text{ A} \]

\[ \therefore -i(t > 0) = \left( \frac{1}{2} \cos 8Mt + \frac{3}{8} \sin 8Mt \right) e^{-6Mt} \text{ A} \]

sol'n: (b) \( \omega_0 > \alpha \Rightarrow \) underdamped