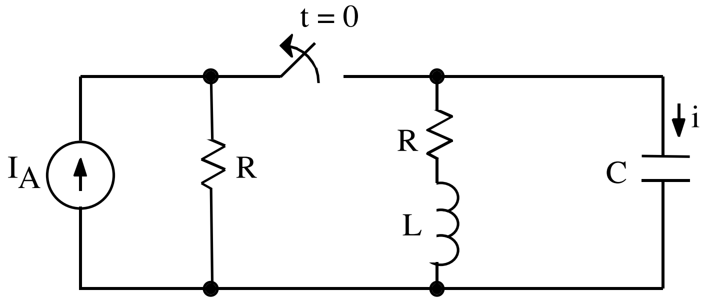
3. (50 points)



$I_A = 1$ A
R = 2400 Ω
L = 200 μH
C = 50 pF

a. After being closed for a long time, the switch is opened at t = 0. Write a numerical time-domain expression for i(t), the current through the capacitance. This expression must not contain any complex numbers.

b. State whether i(t) is underdamped, overdamped, or critically damped.

ans: a) $$i(t > 0) = \left(-\frac{1}{2}\cos 8\text{M}t - \frac{3}{8}\sin 8\text{M}t\right)e^{-6\text{M}t}\text{ A}$$

b) Underdamped.

sol'n: (a) When the switch is open, we have series RLC.

$$\alpha = \frac{R}{2L} = \frac{2.4\text{k}}{2.200\mu} = \frac{1.4k}{400}\text{M} = 6\text{M}/s$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{200\mu\ 50\text{p}} = \frac{1}{10k\ \mu\text{p}} = \frac{1\text{M}\cdot\text{M}\cdot\text{M}}{10\text{k}} = 100\ \text{M}^2/s^2$$

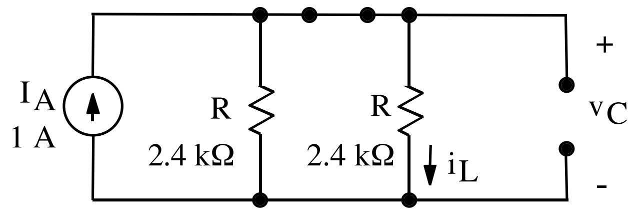
$$\therefore \quad \omega_\text{o} = 10\ \text{M}/s$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{(10\ \text{M})^2 - (6\ \text{M})^2} = 8\ \text{M}/s \qquad \left(6^2 + 8^2 = 10^2\right)$$

Now find initial condition (i.e. i and di/dt, or v and dv/dt at $t = 0^+$).

$$\left.\begin{aligned} i_L(t = 0^+) &= i_L(t = 0^-) \\ v_C(t = 0^+) &= v_C(t = 0^-) \end{aligned}\right\}\text{cannot change instantly}$$

Circuit for $t = 0^-$: L's = wires, C's = open circuits.



$i_L(t=0^-) = \frac{1}{2}$ A (current divider)

$$v_C(t=0^-) = I_A R \,||\, R = I_A \cdot \frac{R}{2} = 1\text{A} \cdot 1.2 \text{ k}\Omega = 1.2 \text{ kV}$$

$$\therefore \quad i_L(t=0^+) = \frac{1}{2}\text{A}, \qquad v_C(t=0^+) = 1.2 \text{ kV}$$

After the switch is open, $i = -i_L$ since C and L are in series.

$\therefore$ Solve for i_L and then change the sign. Note that i_L is the variable in the differential equation for a series RLC. Thus, we know how to find it.

We also need

$$\left.\frac{di_L(t)}{dt}\right|_{t=0^+}.$$

Use V-loop for RLC at $t = 0^+$:

$$L\frac{di_L}{dt} + i_L R - v_C = 0 \text{ V}.$$

Note that at $t = 0^+$, ($i_R = i_L$ since R, L in series),

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = \frac{-i_L(t=0^+)R + v_C(t=0^+)}{L}$$

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = \frac{-\frac{1}{2}A \cdot 2.4 \text{ k}\Omega + 1.2 \text{ k}\Omega}{200 \text{ }\mu\text{H}}$$

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = 0 \text{ A}/s$$

Now use general underdamped solution:

$$i_L(t) = \left(B_1 \cos \omega_d t + B_2 \sin \omega_d t\right)e^{-\alpha t}$$

$$B_1 = i_L(t = 0^+), \qquad -\alpha B_1 + \omega_d B_2 = \left.\frac{di_L}{dt}\right|_{t=0^+} = 0 \text{ A}/s$$

$$\therefore \quad B_1 = \frac{1}{2}\text{A}, \qquad B_2 = \frac{\alpha B_1}{\omega_d} = \frac{6M}{8M}\frac{1}{2}\text{A} = \frac{3}{8}\text{A}$$

$$\therefore \quad -i(t > 0) = \left(\frac{1}{2}\cos 8\text{M}t + \frac{3}{8}\sin 8\text{M}t\right)e^{-6\text{M}t}\text{A}$$

sol'n: (b) $\omega_o > \alpha \Rightarrow$ underdamped