ECE 2260
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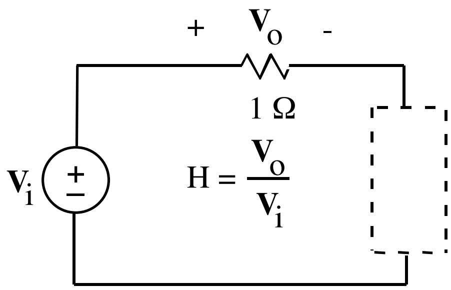
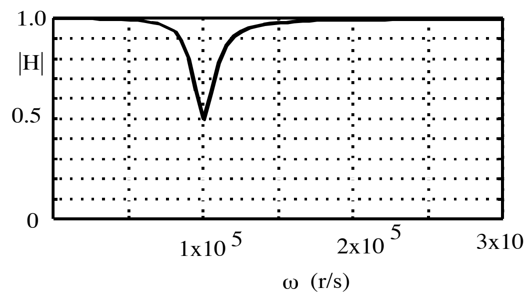
UNIT 2
PRACTICE EXAM SOLUTION: Prob 1

1. (30 points)





Using not more than one each R, L, and C, design a circuit to go in the dashed-line box that will produce the |H| vs. ω shown above, that is:

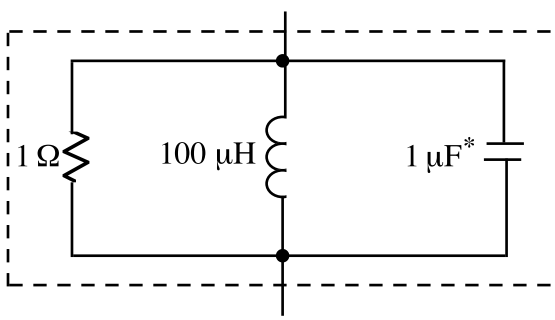
$|H| = 0.5$ at $\omega = 10^5$ rps

$|H| = 1$ at $\omega = 0$

$|H| \to 1$ as $\omega \to \infty$

Specify values of R, L, and C, and show how they would be connected in the circuit. Note that a bandwidth is not specified, and you do not have to satisfy any more than the three requirements specified above.

Ans:



* Any LC = 100 ps is acceptable (if part values are practical).

Sol'n: Given the frequency response plot, we want something resembling a band-reject filter. Since $\mathbf{V}_O$ is measured across R_1, rather than across the dashed box, we want an L and C configuration that has maximum impedance at resonant frequency. Thus, we need an L in parallel with a C inside the dashed box.

If we denote dashed box by z, we have

$$\mathbf{V}_o = \mathbf{V}_i \cdot \frac{R_1}{R_1 + z} \quad \text{(V-divider).} \quad \therefore \quad H(j\omega) \equiv \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_1}{R_1 + z}$$

Note that if

$$z = j\omega L \| \frac{1}{j\omega C} = \frac{\frac{j\omega L}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{L/C}{j\omega L + \frac{1}{j\omega C}}$$

then, at ω_o, we have $j\omega L = -\frac{1}{j\omega C}$.

So $z = \left.\frac{L/C}{0}\right|_{\omega=\omega_o} = \infty$ at $\omega = \omega_o \equiv \frac{1}{\sqrt{LC}}$.

Thus, $\left|H(j\omega)\right|\Big|_{\omega=\omega_o} = \frac{R_1}{R_1 + \infty} = 0$.

We want a value of 1/2, which we'll correct later on. We do have the desired response at high and low frequencies:

At $\omega = 0$,
$$z = \frac{L/C}{j \cdot 0 \cdot L + \frac{1}{j \cdot 0 \cdot C}} = \frac{L/C}{0 + \infty} = 0$$

$$\therefore \quad \left|H(j\omega)\right|\Big|_{\omega=0} = \left|\frac{R_1}{R_1 + z}\right| = \left|\frac{R_1}{R_1}\right| = 1 \quad \checkmark$$

At $\omega \to \infty$,
$$z = \frac{L/C}{j \cdot \infty \cdot L + \frac{1}{j \cdot \infty \cdot C}} = \frac{L/C}{j \cdot \infty \cdot L + 0} = 0$$

$$\therefore \quad \left|H(j\omega)\right|\Big|_{\omega\to\infty} = \left|\frac{R_1}{R_1 + z}\right| = \left|\frac{R_1}{R_1}\right| = 1 \quad \checkmark$$

The remaining problem is to add an R_2 in the dashed box so that $\left|H(j\omega)\right|\Big|_{\omega=\omega_o} = \frac{1}{2}$ instead of zero. For the parallel L and C, we have

$$\left|H(j\omega)\right| = \left|\frac{R_1}{R_1 + R_2}\right| \text{ at } \omega = \omega_o.$$

If we put R_2 in series with the L parallel C, then we would still have $z = R_2 + \infty = \infty$, at $\omega = \omega_o$. Thus, we must try something else.

If we put R_2 in parallel with L parallel C, then we have $z = R_2 \| \infty = R_2$ at $\omega = \omega_o$. This gives

$$\left|H(j\omega)\right| = \left|\frac{R_1}{R_1 + R_2}\right| \text{ at } \omega = \omega_o.$$

We use $R_2 = R_1 = 1\Omega$ to get the required $|H(j\omega)| = 1/2$ at $\omega = \omega_o$.
Now we must verify that we have the correct gain at $\omega = 0$ and $\omega \rightarrow \infty$. For both cases we have

$j\omega L || 1/j\omega C = 0$. The extra R_2 in parallel still gives $z = 0$, as desired.

$\therefore R_2 = 1\Omega$.

Finally, we need $\omega_o = 10^5$ rad/s (dip in plot). Since we have L parallel C even with the addition of R_2, we have the standard resonant frequency:

$$\omega_o = \frac{1}{\sqrt{LC}}.$$

Therefore, we have

$$LC = \frac{1}{\omega_o^2} = \frac{1}{\left(10^5\right)^2} = \frac{100}{10^{12}} = 100\ ps^2.$$

Any $LC = 100\ ps^2$ is acceptable unless the L or C are too large or small to be reasonable. For example, one practical solution is

$C = 1\ \mu F$ and $L = 100\ \mu H$.