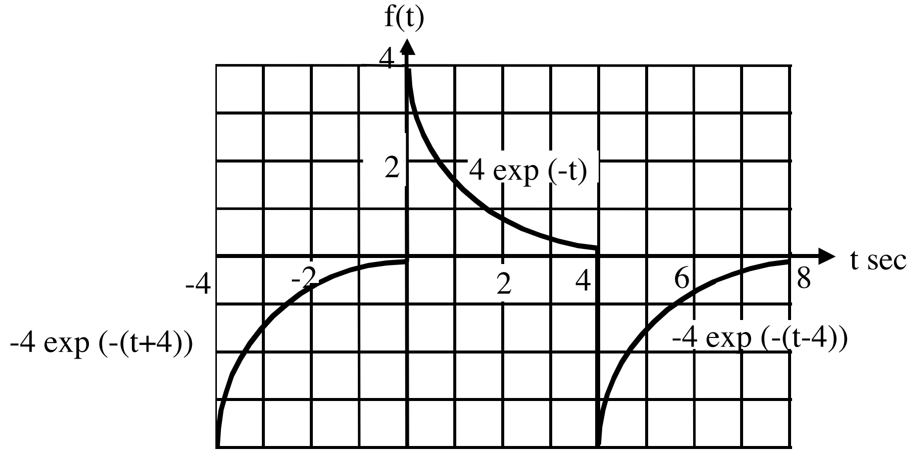
2. (40 points)



Find the coefficients for the Fourier series of the above function. The period of the function is 8 sec. From integral tables or a calculator we have:

$$\int e^{ax}\cos(bx)dx = \frac{e^{ax}\left[a\cos(bx) + b\sin(bx)\right]}{a^2 + b^2}$$

$$\int e^{ax}\sin(bx)dx = \frac{e^{ax}\left[a\sin(bx) - b\cos(bx)\right]}{a^2 + b^2}$$

Ans:

$$a_k = \begin{cases} \dfrac{2\left(1 + e^{-4}\right)}{1 + \left(\dfrac{k\pi}{4}\right)^2} & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$$

$$b_k = \begin{cases} \dfrac{\dfrac{k\pi}{2}\left(1 + e^{-4}\right)}{1 + \left(\dfrac{k\pi}{4}\right)^2} & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$$

Sol'n: Since f(t) has equal area above and below the horizontal axis, we conclude that the DC offset is zero: $a_v = 0$.

Since f(t) is neither even nor odd, we will have nonzero a_k and b_k terms. f(t) does have shift-flip symmetry, however, so even-numbered terms for a_k and b_k will be zero. Furthermore, we need only compute integrals over the first half of the function and double them to find values of a_k and b_k for *k* odd.

We have the following formulas based on shift-flip symmetry:

$$a_k = 2\frac{2}{T}\int_0^{T/2} 4e^{-t}\cos(k2\pi t/T)dt$$

$$b_k = 2\frac{2}{T}\int_0^{T/2} 4e^{-t}\sin(k2\pi t/T)dt$$

Using the integrals given in the problem and T = 8, we have (for *k* odd)

$$a_k = 2\int_0^4 e^{-t}\cos(k\pi t/4)dx = 2\cdot\left.\frac{e^{-t}\left[-\cos(k\pi t/4)+\frac{k\pi}{4}\sin(k\pi t/4)\right]}{(-1)^2+\left(\frac{k\pi}{4}\right)^2}\right|_0^4$$

$$b_k = 2\int_0^4 e^{-t}\sin(k\pi t/4)dx = 2\cdot\left.\frac{e^{-t}\left[-\sin(k\pi t/4)-\frac{k\pi}{4}\cos(k\pi t/4)\right]}{(-1)^2+\left(\frac{k\pi}{4}\right)^2}\right|_0^4$$

Using sin(0) = 0, sin($k\pi$) = 0, and cos(0) = 1, and substituting $t = 4$ for the upper limit, gives

$$a_k = 2\cdot\frac{e^{-4}\left[-\cos(k\pi)\right]+1}{(-1)^2+\left(\frac{k\pi}{4}\right)^2}$$

$$b_k = 2\cdot\frac{\left(-\frac{k\pi}{4}\right)\left[e^{-4}\cos(k\pi)-1\right]}{(-1)^2+\left(\frac{k\pi}{4}\right)^2}$$

We simplify further by noting that cos($k\pi$) = –1 when *k* is odd. With this substitution, we get the final answer.