3. (30 points)

Write the time-domain expression of \( v(t) \) for the first through third harmonics.

Ans: \( v(t) = 1.6 \cos (300k t - 69.4^\circ) \ V \)

Sol'n: Summary of steps expanded upon below:

1. We turn each frequency of \( v_g(t) \) into a phasor.
2. We pass each frequency, \( k\omega_0 \), through the circuit by multiplying by the transfer function \( H(jk\omega_0) \).
3. The result is the phasor for frequency \( k\omega_0 \) in the output signal.
4. We convert the output signal phasor back to the time domain, (through the 1st 3 harmonics).

Note: We only have 1st and 3rd harmonics to worry about since we only have odd harmonics in \( v_g(t) \) and we were only asked to find the output signal's 1st through 3rd harmonics.

1. We turn each frequency of \( v_g(t) \) into a phasor.

\[
k = 1: \quad \frac{16}{\pi} \sin (\omega_0 t) \xrightarrow{\text{P[1]}} -j \frac{16}{\pi} \text{ or } \frac{16}{\pi} \angle -90^\circ
\]

\[
k = 3: \quad \frac{16}{3\pi} \sin (3\omega_0 t) \xrightarrow{\text{P[1]}} -j \frac{16}{3\pi} \text{ or } \frac{16}{3\pi} \angle -90^\circ
\]

2. We pass each frequency, \( k\omega_0 \), through the circuit by multiplying by the transfer function \( H(jk\omega_0) \).
\[ H(j\omega) = \frac{V}{V_g} = \frac{j\omega + \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} \quad \text{(V-divider)} \]

\[ k = 1: \]

\[ H(j\omega_0) = \frac{(j\omega_0)^2 LC + 1}{j\omega_0 RC + (j\omega_0)^2 LC + 1} \]

\[ H(j\omega_0) = \frac{-(100k)^2 0.1m \mu + 1}{j100k 10k \mu - (100k)^2 0.1m \mu + 1} \]

\[ H(j\omega_0) = \frac{-100^2 \cdot 0.1m + 1}{j1k - 100^2 0.1m + 1} = \frac{0}{j1k} = 0 \]

In this case, the L and C together behave like a wire at frequency \( \omega_0 \). The voltage drop across the wire is zero, as is the circuit output. Consequently, \( H(j\omega_0) = 0 \).

Note: In this case, the \( \omega_0 \) for the \( v_g(t) \) Fourier series happens to be the same as the center (or resonant, or characteristic) frequency of the L and C:

\[ \omega_0 = \sqrt{\frac{1}{LC}} \]

Typically, the \( \omega_0 \) for the Fourier series is different from the \( \omega_0 \) for an L and C.

\[ k = 3: \]

\[ H(j3\omega_0) = \frac{-3^2 + 1}{j3k - 3^2 + 1} = \frac{-8}{j3k - 8} \]

Note: \((j3\omega_0)^2 LC\) is 3^2 since it was 1^2 for \( \omega_0 \).

\[ H(j3\omega_0) = \frac{8}{8 - j3k} = \frac{1}{1 - j3k/8} = \frac{1}{1 - j375} = 2.67 \, m\Omega \angle 89.85^\circ \]

3. The result is the phasor for frequency \( k\omega_0 \) (where \( k = 3 \)) in output signal.

Input phasor is

\[ V_{i3} = -j \frac{16}{3\pi} \, V = \frac{16}{3\pi} \angle -90^\circ \, V. \]

Output phasor is

\[ V_{o3} = V_{i3} H(j3\omega_0) = \frac{16}{3\pi} \angle -90^\circ \cdot 2.67 \angle 89.85^\circ \, mV. \]

\[ V_{o3} = 4.53 \angle -0.15^\circ \, mV \]
4. We convert the output signal phasor back to the time domain, (through the 1st 3 harmonics, of which only the third harmonic is nonzero).

\[ v_{o3} = 4.53\cos(300\, kt - 0.15^\circ) \text{ mV} \]