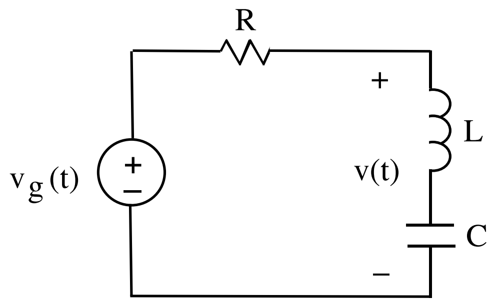
3. (30 points)



$\omega_o = 10^5 \frac{\text{rad}}{\text{sec}}$

L = 0.1 mH

C = 1 μF

R = 10 kΩ

$$v_g(t) = \frac{16}{\pi} \sum_{n \text{ odd}}^{\infty} \frac{1}{n} \sin(n\omega_o t)$$

Write the time-domain expression of v(t) for the first through third harmonics.

Ans: v(t) = 1.6 cos (300k *t* - 69.4°) V

Sol'n: Summary of steps expanded upon below:

1. We turn each frequency of $v_g(t)$ into a phasor.
2. We pass each frequency, $k\omega_o$, through the circuit by multiplying by the transfer function $H(jk\omega_o)$.
3. The result is the phasor for frequency $k\omega_o$ in the output signal.
4. We convert the output signal phasor back to the time domain, (through the 1st 3 harmonics).

Note: We only have 1st and 3rd harmonics to worry about since we only have odd harmonics in $v_g(t)$ and we were only asked to find the output signal's 1st through 3rd harmonics.

1. We turn each frequency of $v_g(t)$ into a phasor.

$$k = 1: \quad \frac{16}{\pi}\sin(\omega_o t) \xrightarrow{\textbf{P[ ]}} -j\frac{16}{\pi} \text{ or } \frac{16}{\pi}\angle -90^\circ$$

$$k = 3: \quad \frac{16}{\pi}\frac{1}{3}\sin(3\omega_o t) \xrightarrow{\textbf{P[ ]}} -j\frac{16}{3\pi} \text{ or } \frac{16}{3\pi}\angle -90^\circ$$

2. We pass each frequency, $k\omega_o$, through the circuit by multiplying by the transfer function $H(jk\omega_o)$.

$$H(j\omega) \equiv \frac{V}{V_g} = \frac{j\omega + \dfrac{1}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}} \text{ (V-divider)}$$

k = 1:

$$H(j\omega_o) = \frac{(j\omega_o)^2 LC + 1}{j\omega_o RC + (j\omega_o)^2 LC + 1}$$

$$H(j\omega_o) = \frac{-(100k)^2 0.1m\ 1\mu + 1}{j100k\ 10k\ 1\mu - (100k)^2 0.1m\ 1\mu + 1}$$

$$H(j\omega_o) = \frac{-100^2 \cdot 0.1m + 1}{j1k - 100^2 0.1m + 1} = \frac{0}{j1k} = 0$$

In this case, the L and C together behave like a wire at frequency ω_o. The voltage drop across the wire is zero, as is the circuit output. Consequently, $H(j\omega_o) = 0$.

Note: In this case, the ω_o for the $v_g(t)$ Fourier series happens to be the same as the center (or resonant, or characteristic) frequency of the L and C:

$$\omega_o = \sqrt{\frac{1}{LC}}$$

Typically, the ω_o for the Fourier series is different from the ω_o for an L and C.

k = 3:

$$H(j3\omega_o) = \frac{-3^2 + 1}{j3\,\text{k} - 3^2 + 1} = \frac{-8}{j3\,\text{k} - 8}$$

Note: $(j3\omega_o)^2 LC$ is 3^2 since it was 1^2 for ω_o.

$$H(j3\omega_o) = \frac{8}{8 - j3\,\text{k}} = \frac{1}{1 - \dfrac{j3\,\text{k}}{8}} = \frac{1}{1 - j375} = 2.67\ \text{m}\angle 89.85^\circ$$

3. The result is the phasor for frequency $k\omega_o$ (where $k = 3$) in output signal.

   Input phasor is

   $$V_{i3} = -j\frac{16}{3\pi}\ \text{V} = \frac{16}{3\pi}\angle -90^\circ\ \text{V}.$$

   Output phasor is

   $$V_{o3} = V_{i3} H(j3w_o) = \frac{16}{3\pi}\angle -90^\circ \cdot 2.67\ \angle 89.85\ \text{mV}.$$

   $$V_{o3} = 4.53\angle -0.15^\circ\ \text{mV}$$

4. We convert the output signal phasor back to the time domain, (through the 1st 3 harmonics, of which only the third harmonic is nonzero).

   $v_{o3} = 4.53\cos(300\text{ k}t - 0.15°)$ mV