3. (40 points)

\[ i(t) = \frac{3}{4} \sqrt{2} \cos (2kt - 135^\circ) A \]

b) \[ V_1 = 46.7 \angle 58.8^\circ V \]
**sol'n: (a)** We assume an ideal transformer since we are only given the turns ratio \( N_1/N_2 \). We want to draw the circuit in the \( s \)-domain with labels for \( I_1, I_2, V_1, \) and \( V_2 \).

\[
\omega = 2k \quad \text{from} \quad V_s = 100 \cos (2k\cdot t) \text{V}
\]

\[
\frac{1}{j\omega C} = \frac{1}{j2k \cdot 5\mu} \Omega = -\frac{j}{10m} \Omega = -j100\Omega
\]

Our circuit diagram in the \( s \)-domain:

![Circuit Diagram](image)

Note: We measure \( V_1 \) and \( V_2 \) with plus signs at dots on transformer.

- \( I_1 \) (primary side) flows into dotted terminal.
- \( I_2 \) (secondary side) flows out of dotted terminal.

For the above definitions of \( I_1, I_2, V_1, \) and \( V_2 \), we have ideal transformer equations without minus signs:

\[
\frac{V_1}{N_1} = \frac{V_2}{N_2} \quad I_1N_1 = I_2N_2
\]

Now we write equations for mesh (current) loops. (We could also use the node-voltage method.)

We observe that \( I = I_2 \) in the top loop, and since \( I_2 \) is flowing on the outer edge of the circuit, (where there is no circuit on the other side to cause a summation of mesh currents through components), we see that \( I \) is also the mesh current for the top loop.

The current mesh equation for the top loop is (the sum of V drops around the loop):

\[
(1) \quad V_2 - I \cdot 10\Omega + V_1 - I (-j100\Omega) = 0 \text{V}
\]

Note: \( I \) must also flow up through C. What \( I \) goes down, must come up. (Otherwise, we would accumulate charge in the bottom half of the circuit.)

The mesh current for the bottom loop will be \( I + I_1 \). This current is flowing on the outside edge of the circuit in the bottom loop.
Our mesh loop equation for the bottom (i.e. sum of V drops around loop) is:

\[-V_1 - (I + I_1)40\Omega - 100\angle0^\circ = 0V\]

or

\[(2) \quad -V_1 + (I + I_1)40\Omega + 100\angle0^\circ = 0V\]

Now we use the ideal transformer equations to eliminate all but two unknowns:

\[V_1 = \frac{N_1}{N_2}V_2, \quad I_1 = \frac{N_2}{N_1}I_2 = \frac{N_2}{N_1}I = \frac{I}{2}\]

Substituting these into Eq. 1 and Eq. 2 gives two equations in two unknowns:

\[(1') \quad V_2 - I \cdot 10\Omega + 2V_2 - I(-j100\Omega) = 0V\]

\[(2') \quad 2V_2 + \left(\frac{I}{2}\right)40\Omega + 100\angle0^\circ = 0V\]

Now we solve for I. From Eq. 2':

\[V_2 = -\left(\frac{3}{2}I \cdot 40\Omega + 100\right) = -30 \cdot I - 50V\]

Eq. 1' rearranged is

\[3V_2 + (-10 + j100)I = 0V\]

By substituting for \(V_2\), and doing the algebra, we find I:

\[3(-30 \cdot I - 50) + (-10 + j100)I = 0V\]

\[(-100 + j100)I - 150 = 0V\]

\[I = \frac{150}{-100 + j100}A = \frac{3}{-2 + j2}\ A = -\frac{3}{2} \frac{1}{1 - j} \ A\]

\[I = -\frac{3}{4}\sqrt{2} \angle45^\circ \ A\]

\[I = \frac{3}{4}\sqrt{2} \angle -180^\circ + 45^\circ \ A\]

\[I = \frac{3}{4}\sqrt{2} \angle -135^\circ \ A\]

\[∴ \quad i(t) = \frac{3}{4}\sqrt{2} \cos (2kt - 135^\circ)A\]
sol'n: (b) We use the idea of reflected impedance for a linear transformer:

The formula for reflected impedance with a linear transformer is

\[ Z_r = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L} \]

Here, we have \( R_2 = 0 \Omega, j\omega L_2 = j20 \Omega, \) and \( Z_L = 35 \Omega + j40 \Omega. \)

\( j\omega M = j15 \Omega \) is the mutual inductance. \( \therefore \omega M = 15 \Omega \)

\[ Z_r = \frac{225}{j20 + 35 + j40 \Omega} + \frac{225}{35 + j60 \Omega} = \frac{45}{7 + j12} \Omega \]

Using the equivalent model for primary side, as shown above, we have

\[ V_1 = \frac{j30 \Omega + Z_r}{50 + j30 + Z_r} \cdot 100 \angle 0^\circ V \]

\[ V_1 = \frac{j30 + \frac{45}{193}(7 - j12)}{50 + \frac{45}{193}(7 - j12)} \cdot 100 \angle 0^\circ V \]

We can factor out a 5 from top and bottom:

\[ V_1 = \frac{\frac{j(193)30}{193} + \frac{45(7 - j12)}{193}}{\frac{10}{6} + \frac{45(7 - j12)}{9}} \cdot \frac{20}{1993 + j1050} \cdot 20 \Omega \]

Evaluating the expression and converting to polar form gives

\[ V_1 = 46.7 \angle 58.8^\circ V. \]