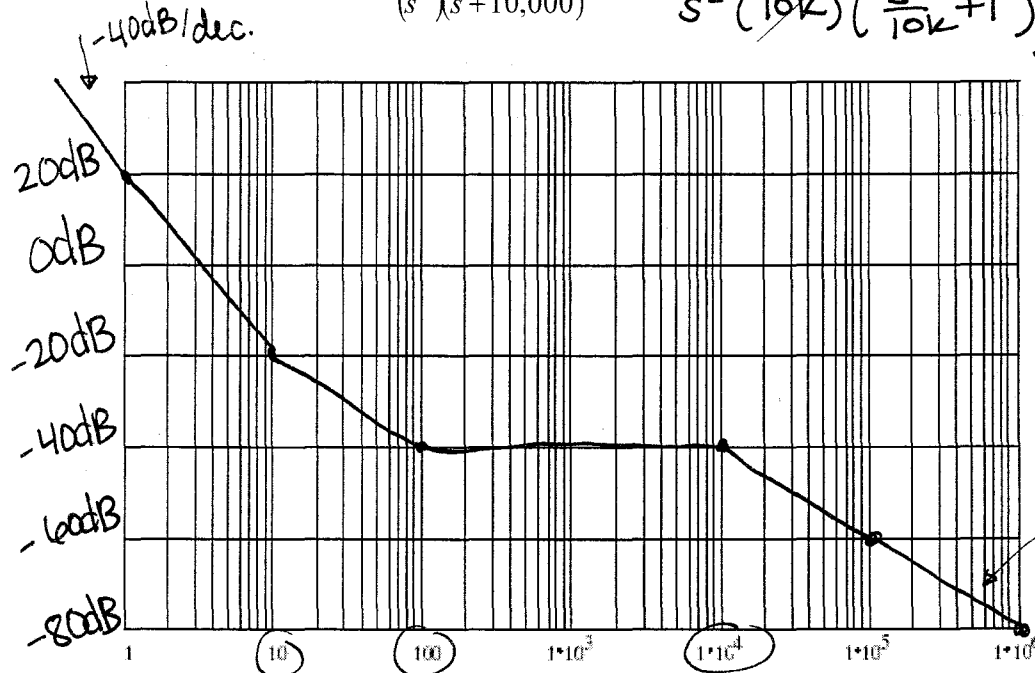
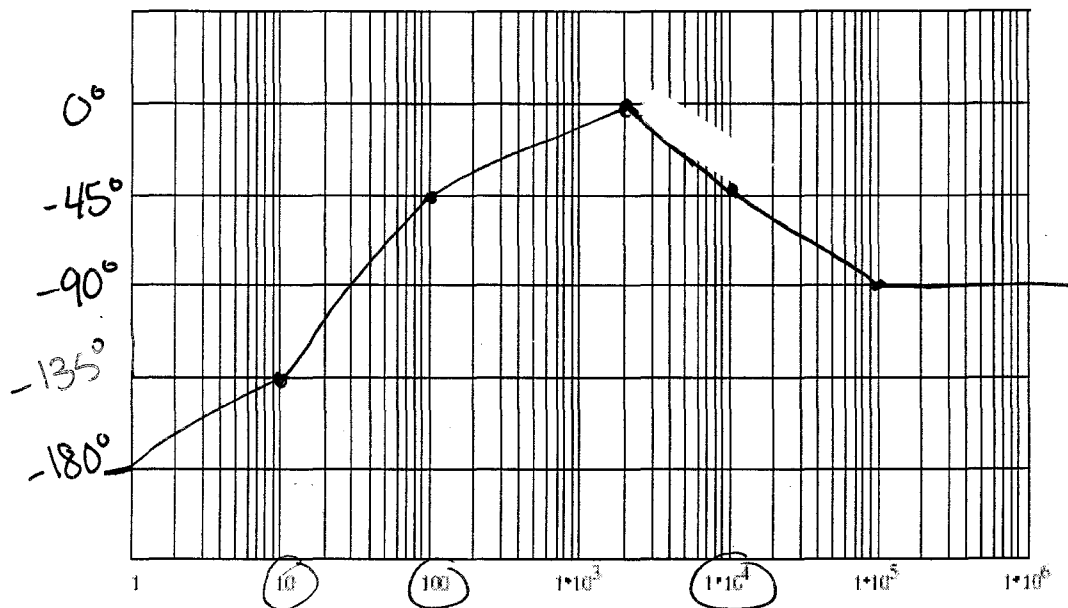


Examples #2:

$$\text{Transfer Function} = \frac{(100)(s+100)(s+10)}{(s^2)(s+10,000)} = \frac{100(100)\left(\frac{s}{100}+1\right)(10)\left(\frac{s}{10}+1\right)}{s^2(10k)\left(\frac{s}{10k}+1\right)}$$



$20 \log(10) = 20\text{dB}$
 phase $\rightarrow -180^\circ$
 start slope \rightarrow
 $+2(-20\text{dB/dec})$
 $= -40\text{dB/dec}$
 at $\omega=1$



-20dB/dec.

$\omega=10$: -zero: $+20\text{dB/dec}$: $+45^\circ$ ($\omega=1 \rightarrow 10, 10 \rightarrow 100$)
 $\omega=100$: -zero: $+20\text{dB/dec}$: " ($\omega=10 \rightarrow 100, 100 \rightarrow 1k$)
 $\omega=10k$: -pole: -20dB/dec : -45° ($\omega=1k \rightarrow 10k, 10k \rightarrow 100k$)

Example #3:

$$\text{Transfer Function} = \frac{5 \times 10^4 s}{s^2 + 505s + 2500}$$

$$\frac{5 \times 10^4 s}{5(\frac{s}{5} + 1)(500)(\frac{s}{500} + 1)} = \frac{20s}{(\frac{s}{5} + 1)(\frac{s}{500} + 1)}$$

$$20 \log(20) = 26 \text{ dB} @ \omega = 1$$

phase $\Rightarrow 90^\circ$
start

Slope start $\Rightarrow +20 \text{ dB/dec}$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

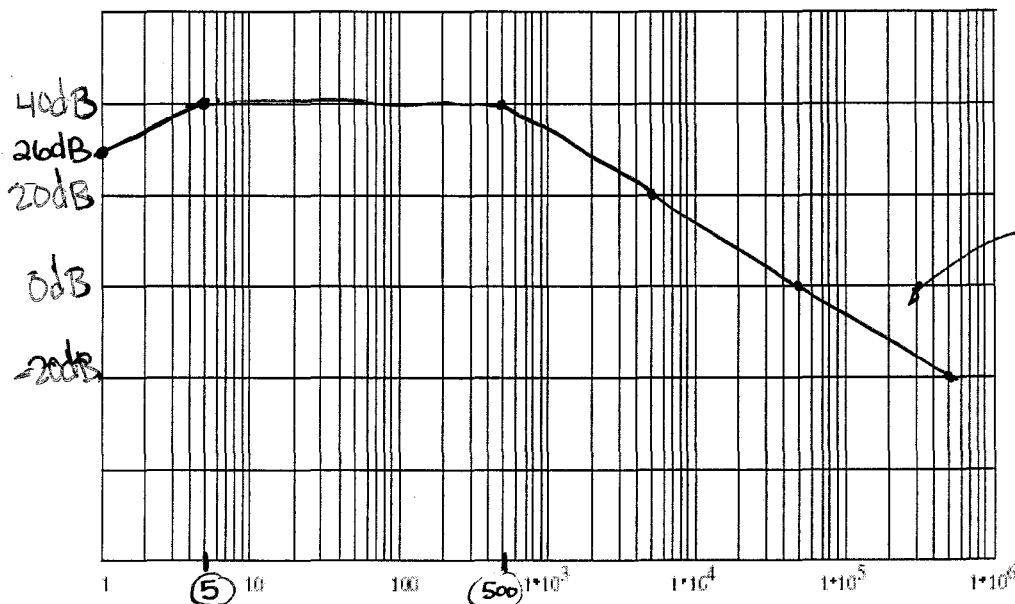
$$s = \frac{-505 \pm \sqrt{505^2 - 4(2500)}}{2}$$

$$= -5, -500$$

$$(s + 5)(s + 500)$$

$\omega = 5$: -pole: -20 dB/dec : -45°
($\omega = 0.5 \rightarrow 5, 5 \rightarrow 50$)

$\omega = 500$: -pole: -20 dB/dec : -45°
($\omega = 50 \rightarrow 500, 500 \rightarrow 5k$)

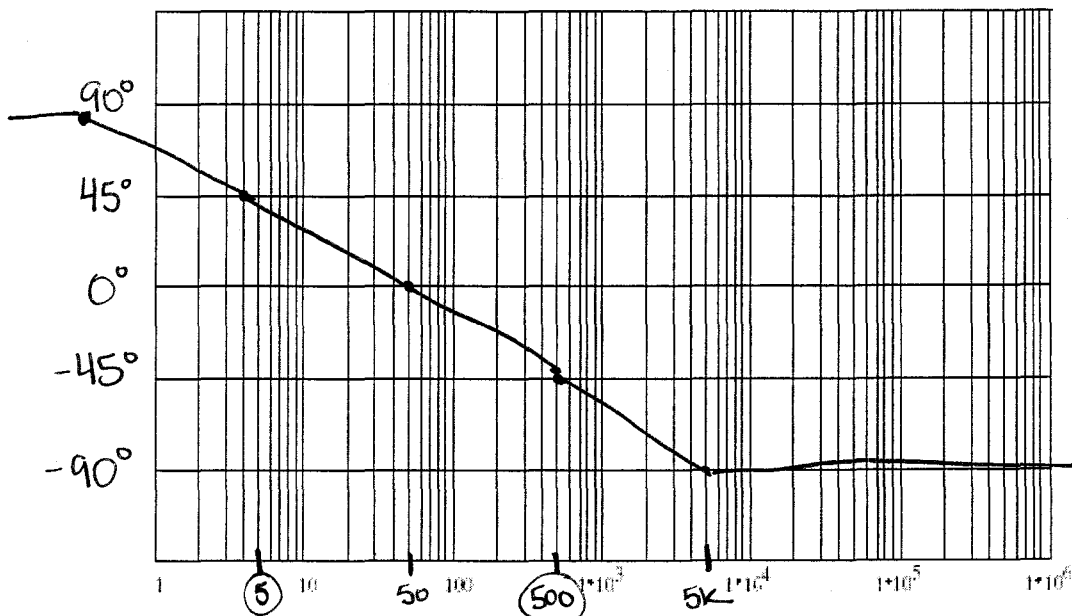


-20 dB/dec
 $\omega = 10$:

$$\frac{20(10)}{\sqrt{(\frac{10}{5})^2 + 1} \sqrt{(\frac{10}{500})^2 + 1}} \approx 39 \text{ dB}$$

$\omega = 5$:

$$\frac{20(5)}{\sqrt{(\frac{5}{5})^2 + 1} \sqrt{(\frac{5}{500})^2 + 1}} \approx 37 \text{ dB}$$

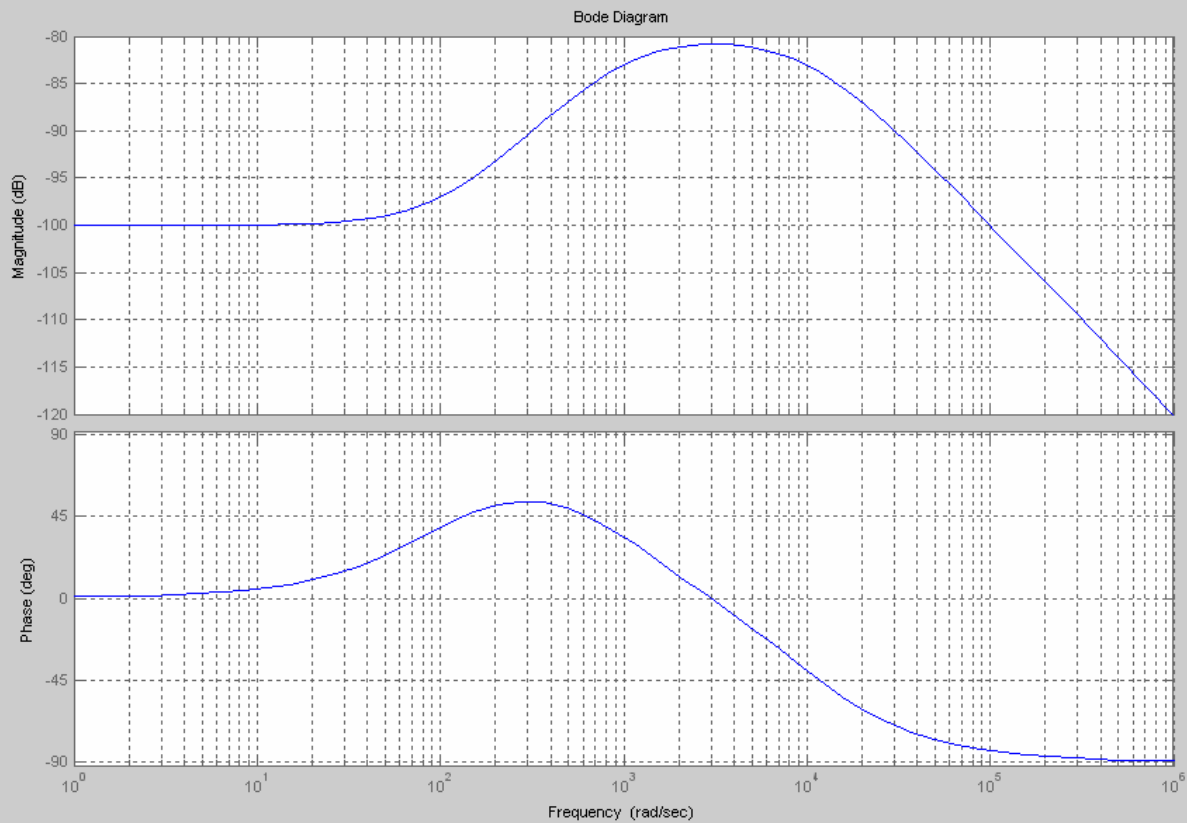


PROBLEM SESSION #1 PROBLEMS

1. Calculate Bode Plots of the following:

$$(a) H(s) = \frac{(s + 100)}{(s + 10^3)(s + 10^4)}$$

- Start value: $H(0) = 100/(10^3 \cdot 10^4) = 10^{-5} \Rightarrow 20 \log_{10}(10^{-5}) = -100 \text{ dB}$
- Critical frequencies:
 - $\omega = 100$ – (negative zero) $\Rightarrow +20 \text{ dB/dec} / +45^\circ \text{ slope/dec}$ (over 2 decades $10 < \omega < 1,000$)
 - $\omega = 1,000$ – (negative pole) $\Rightarrow -20 \text{ dB/dec} / -45^\circ \text{ slope/dec}$ (over 2 decades $100 < \omega < 10,000$)
 - $\omega = 10,000$ – (negative pole) $\Rightarrow -20 \text{ dB/dec} / -45^\circ \text{ slope/dec}$ (over 2 decades $1,000 < \omega < 100,000$)



2. Calculate the Bode plot for the following:

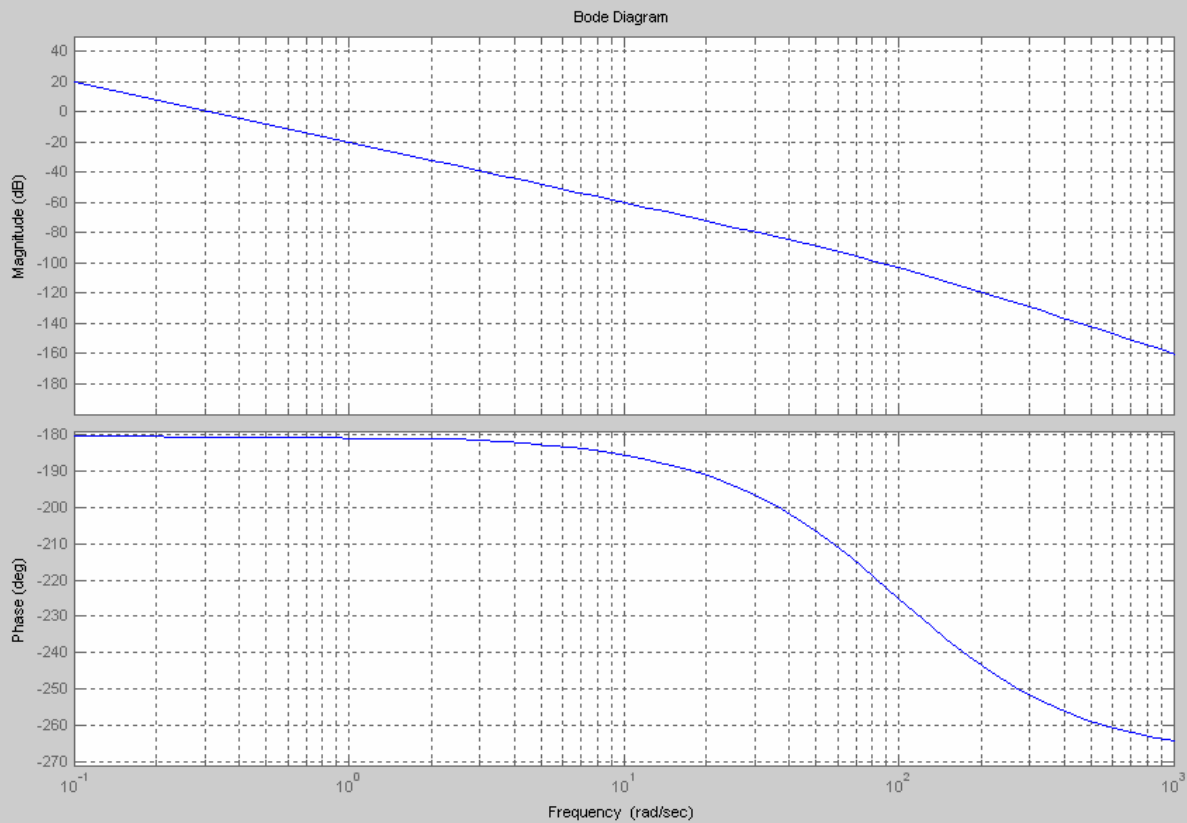
$$H(s) = \frac{10}{s^2(s+100)}$$

(a) $n=-2$ (the number of poles or zeros at the origin – 2 poles at the origin)

- gain: $K=H(s)*s^2|_{s=0} = 10/100 = .1 \Rightarrow 20\log_{10}(.1*1^{-2}) = -20\text{dB}$
 - choose $\omega_{\text{start}} = 0.1$ and you get $20\log_{10}(.1*1^{-2}) = 20\text{dB}$
- phase: $K>0, n*90^\circ = -2*90^\circ = -180^\circ$

(b) critical frequencies:

- $\omega=0$ – (pole at origin) $\Rightarrow -40\text{dB/dec}/ -180^\circ$ start
- $\omega=1,00$ – (negative pole) $\Rightarrow -20\text{dB/dec}/ -45^\circ$ slope/dec (over 2 decades $10<\omega<1,000$)



4. Use Matlab for each function listed below to obtain the Bode Plot. Sketch the Bode plots using a straight-line approximation (procedures described in class) and compare the two:

a.
$$H(s) = \frac{s + 2}{(s + 10)(s + 100)}$$

b.
$$H(s) = \frac{10s}{(s + 1)(s + 10)}$$

c.
$$H(s) = \frac{10(s + 1)}{s^2 (s^2 - 2s + 100)}$$

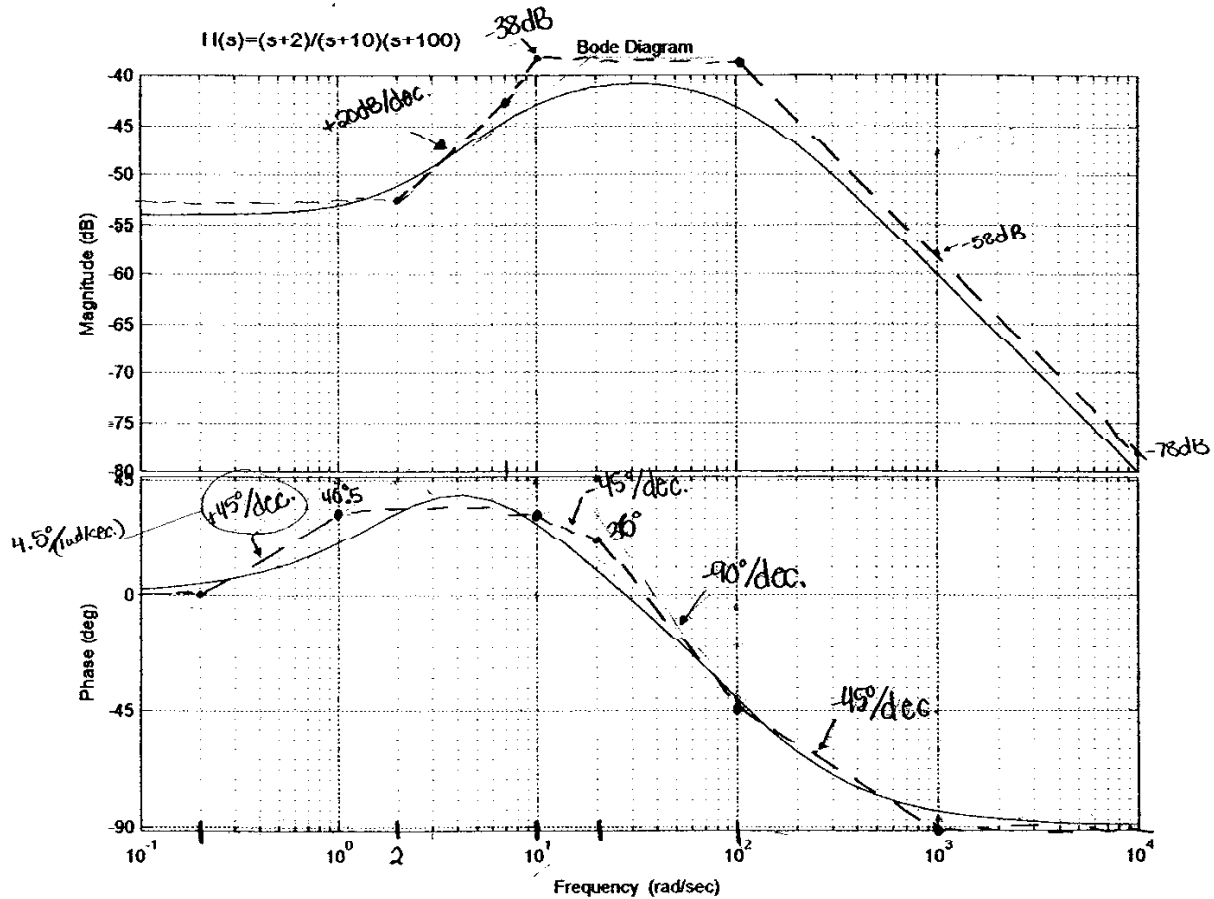
5. The transfer function of a system is given by

④ a. Step 1: $H(0) = \frac{2}{(10)(100)} = 2m \Rightarrow 20 \log_{10}(2m) = -53.98$

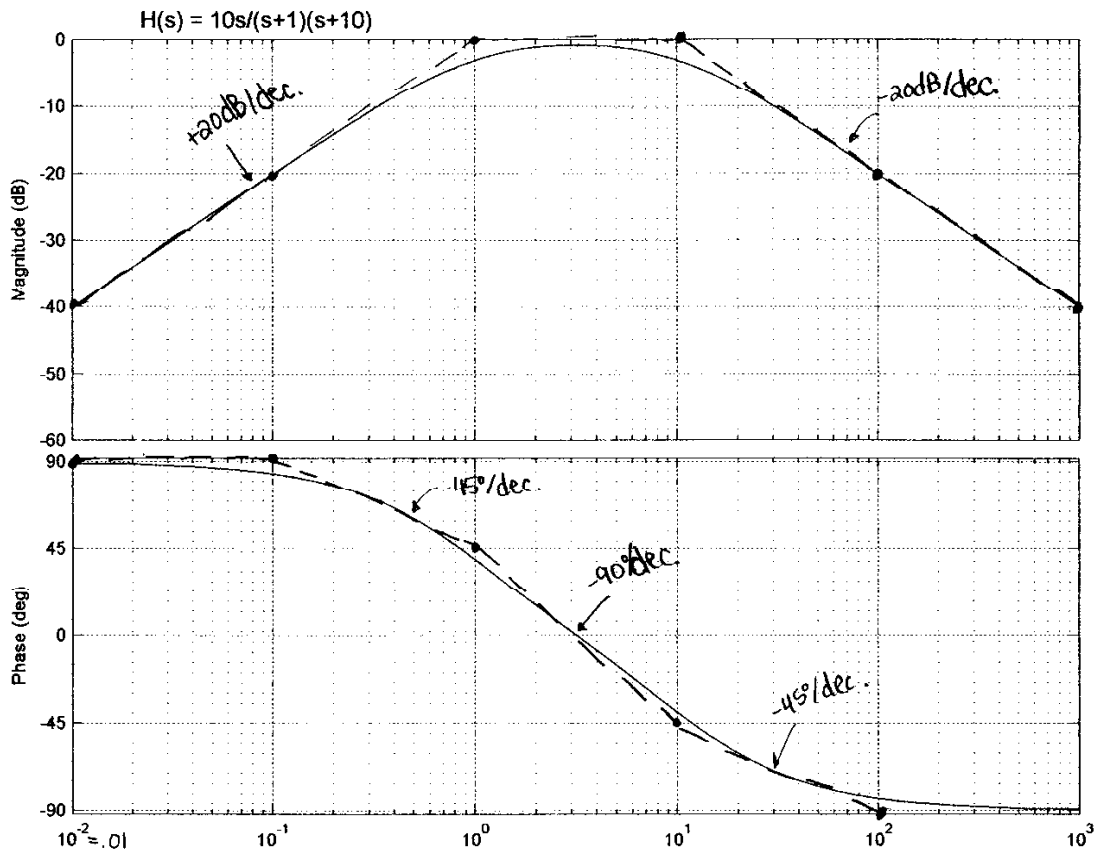
Step 2: since $H(0) > 0$, phase = 0°
 critical freq: ① $\omega = 2$ (negative zero): $+20 \text{ dB/dec.}$, $+45^\circ/\text{s}$

② $\omega = 10$ (negative pole): -20 dB/dec. , $-45^\circ/\text{slope}$

③ $\omega = 100$ (negative pole): -20 dB/dec. , $-45^\circ/\text{slope}$



- ④ b. step 1: (starting value) remove the zero from the transfer function and find the DC gain: $K = \lim_{s \rightarrow 0} H(s) \cdot \frac{1}{s} = \frac{10}{1(10)} = 1 \Rightarrow 0\text{dB}$
- The start value will be $20 \log_{10}(|K| \cdot \omega_{\text{start}}^n)$ where n is (+) for a zero, negative for a pole. $n = \#$ of poles or zeros at $s=0$.
- ↳ If $\omega_{\text{start}} = .01 \Rightarrow 20 \log_{10}(|1| \cdot .01) = -40\text{dB}$. The line will be $n \times 20\text{dB/dec}$
- Draw a horizontal line at $\sigma + n \cdot 90^\circ$ if $K > 0$ or $180^\circ + n \cdot 90^\circ$ if $K < 0$
 - $K = 1 \therefore \text{phase} \rightarrow 90^\circ$ ($n=1$ for our case)



critical frequency's @ $\omega = 1$ (negative pole): $-20\text{dB/dec.}, -45^\circ/\text{dec.}$
 $\omega = 10$ (negative pole): $-20\text{dB/dec.}, -45^\circ/\text{dec.}$