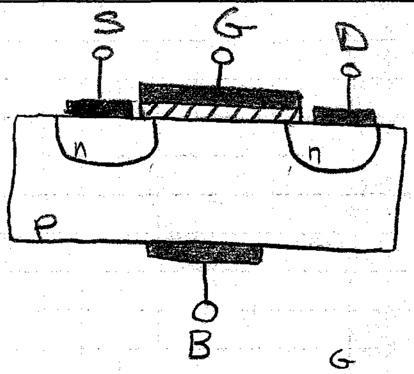
ECE2280 | HW#4 Solution

1) (a)

NMOS

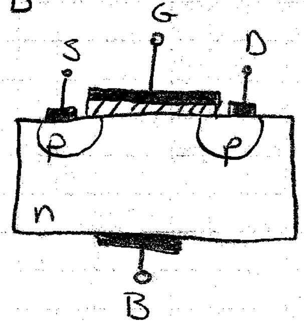


- ■ – Metal (Al)
- ▨ – Oxide (SiO_2)
- □ – Semiconductor (Si)

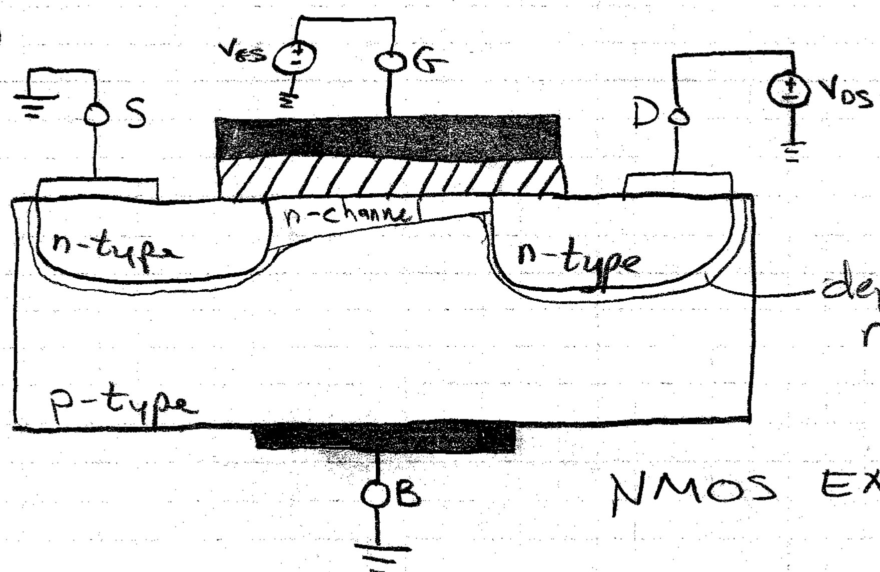
S ≡ Source
G ≡ Gate
D ≡ Drain
B ≡ Body

or

PMOS



(b)



NMOS EXAMPLE

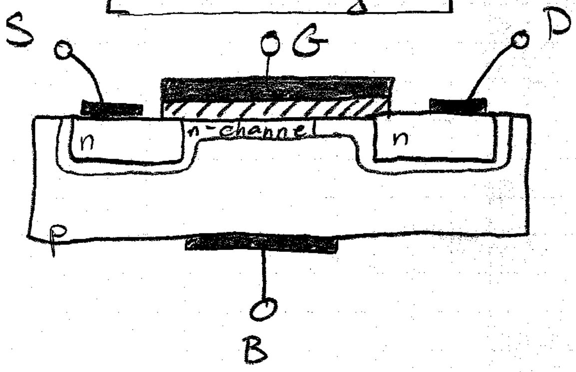
Current flows in the MOSFET when:

1. $V_{GS} > V_t$
2. $V_{DS} \neq 0\,V$

Note: If $V_{DS} < 0$, then the source becomes the drain and the drain becomes the source since the MOSFET as shown is symmetrical (not true for discrete transistors since the body is usually tied to the source).

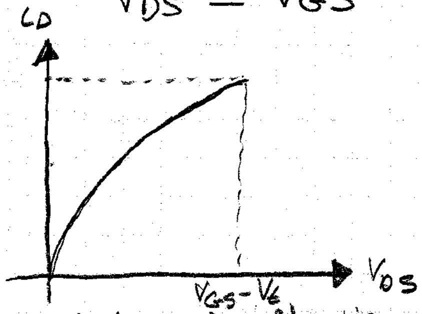
(c)

Triode Region



$V_{GS} > V_t$

$V_{DS} \leq V_{GS} - V_t$



Induced channel increases at nearly the same rate in all directions with increased V_{DS} in this region. The I-V relationship is shown on the graph. It is characterized by the following equation.

$$i_D = k_n' \frac{W}{L}\left[(V_{GS}-V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

where: $k_n' = \mu_n C_{ox}$

$\mu_n \equiv$ electron mobility of semiconductor material

$C_{ox} \equiv$ gate capacitance per unit gate area ($C_{ox} = \varepsilon_{ox}/t_{ox}$)

$t_{ox} \equiv$ oxide thickness

$\varepsilon_{ox} \equiv$ oxide permittivity

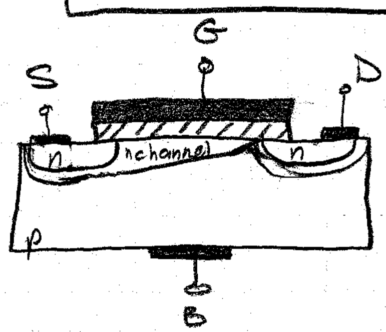
$\frac{W}{L} \equiv$ aspect ratio

$W \equiv$ channel width

$L \equiv$ channel length

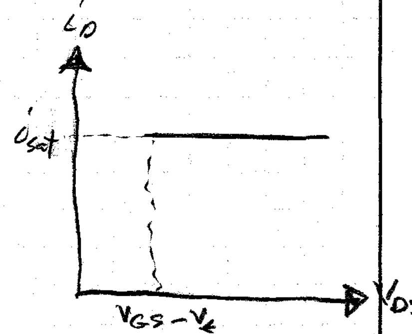
$V_t \equiv$ threshold voltage

Saturation Region



$V_{GS} > V_t$

$V_{DS} \geq V_{GS} - V_t$



$$i_D = \frac{1}{2}k_n' \frac{W}{L}(V_{GS}-V_t)^2$$

$$i_{sat} = \frac{1}{2}k_n' \frac{W}{L}(V_{GS}-V_t)^2$$

Induced Channel is uneven and "pinched off" stopping the current from increasing with the voltage. V_{DS}. Note: It still increases by a small amount. See page 254, figure 4.16 of the textbook.

2) (a) Figure 4-6 is demonstrating the I-V-relationship of a MOSFET for a fixed value of V_{GS}.

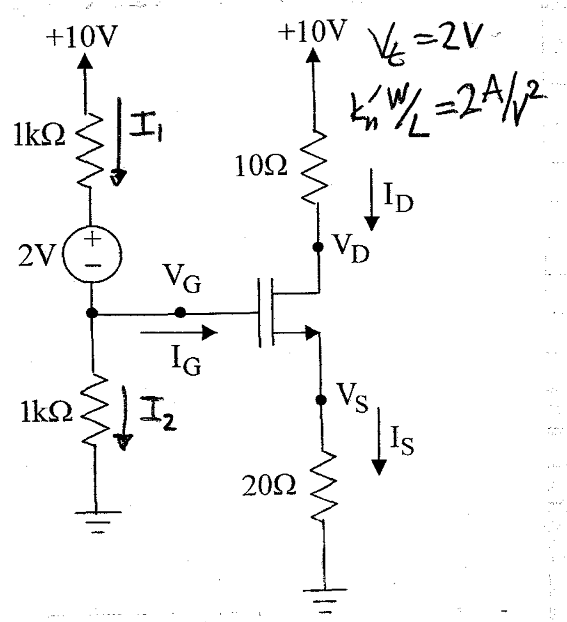
Figure 4-11 is demonstrating the same thing for various values of V_{GS}.

Fig. 4-6	Fig 4-11
i_D varies	i_D varies
V_{DS} varies	V_{DS} varies
V_{GS} constant	V_{GS} varies

(b) This is the saturation region when V_{DS} is held constant and V_{GS} is varied.

(c) As shown in the figure for question #1a, a PMOS is built in an n-type substrate so that it must induce a p-type channel. An NMOS is built in a p-type substrate so that it must induce an n-type channel.

3) Find V_D, V_G, V_S, I_D, I_S, I_G, I_1, I_2
Neglect Channel Length modulation.



Assume $I_G = 0$, then

$$I_1 = I_2 = \frac{10V - 2V}{2k\Omega} = 4mA$$

$$V_G = (4mA)(1k\Omega) = 4V$$

$$V_S = I_S(20\Omega)$$

$$V_D = 10V - I_D(10\Omega)$$

Assume Triode

$$I_D = 2\frac{A}{V^2}\left[(V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

$V_{GS} = V_G - V_S$
$V_{DS} = V_D - V_S$

$$I_S = \frac{V_S}{20\Omega}$$

$$I_D = \frac{10V - V_D}{10\Omega}$$

$$I_S = I_D$$

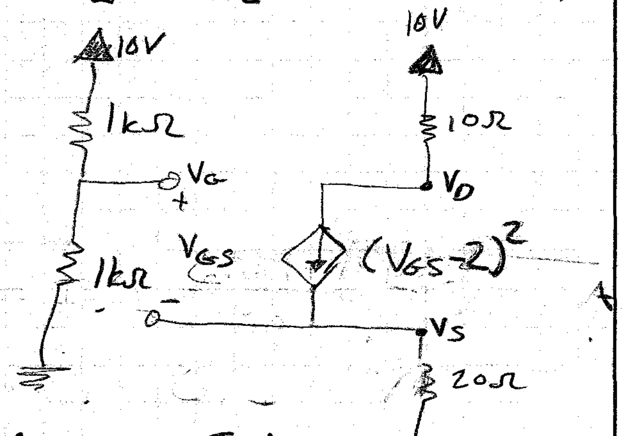
$I_S = I_D \Rightarrow \frac{V_S}{20\Omega} = \frac{10V - V_D}{10\Omega} \Rightarrow V_D = 10V - \frac{1}{2}V_S$

$\Rightarrow V_{DS} = 10V - \frac{3}{2}V_S$

Substitute into triode equation.

$$\frac{V_S}{20\Omega} = 2\frac{A}{V^2}\left[(2V - V_S)(10V - \tfrac{3}{2}V_S) - \tfrac{1}{2}(10V - \tfrac{3}{2}V_S)^2\right]$$

$$V_S = 40\frac{1}{V}\left[200V^2 - 3V_S - 10V_S + \tfrac{3}{2}V_S^2 - \tfrac{1}{2}(100V^2 - 30V_S + \tfrac{9}{4}V_S^2)\right]$$



$15V_S^2 + 79V_S - 1200 = 0$

$V_S^2 + \frac{79}{15}V_S - 80 = 0$

$V_S = 6.691V$

$V_{GS} = V_G - V_S = 4V - 6.691V$

$V_{GS} < 0$? wrong! Assume Saturation.

$I_D = (V_{GS} - 2)^2 = (2 - V_S)^2 = \frac{V_S}{20}$

$\Rightarrow V_S^2 - 4V_S - \frac{V_S}{20} + 4 = 0$

$\Rightarrow V_S^2 - \frac{81}{20}V_S + 4 = 0$

$V_S = 2.3422V$ or $1.7078V$

$V_{GS} = 4V - 2.34 = 1.66V < V_t = 2V$ wrong!

$V_{GS} = 4V - 1.71 = 2.29V > V_t = 2V$ right!

$I_D = I_S = \frac{V_S}{20\Omega} = \frac{1.71V}{20\Omega} = 85.5mA$

$V_D = 10V - 10\Omega(85.5mA) = 9.15V$

Answers:

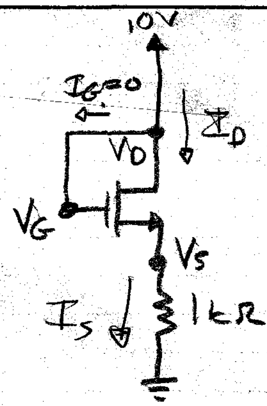
The MOSFET IS IN SATURATION: $V_{GS} > V_t$, $V_{DS} \geq V_{GS} - V_t$

$V_{GS} = 4V - 1.71V = 2.29V > V_t$ ✓

$V_{DS} = 9.15V - 1.71V = 7.44V > V_{GS} - V_t$ ✓

Voltages:	Currents:
$V_S = 1.71V$	$I_S = I_D = 85.5mA$
$V_D = 9.15V$	$I_G = 0A$
$V_G = 4V$	$I_1 = I_2 = 4mA$

(4) Find V_S, $\lambda = 0$, $|V_t| = 1$, $k_n' \frac{W}{L} = 1mA/V^2$



$V_G = V_D \Rightarrow V_{DS} > V_{GS} - |V_t| \Rightarrow$ MOSFET Sat.

$$I_D = I_S = \frac{k_n'}{2}\frac{W}{L}(V_{GS} - |V_t|)^2$$

$$I_S = \frac{1mA}{2}/V^2 (9V - V_S)^2$$

$$I_S = \frac{V_S}{1k\Omega}$$

$$\therefore 2V_S = (9 - V_S)^2$$

$$V_S^2 - 20V_S + 81 = 0$$

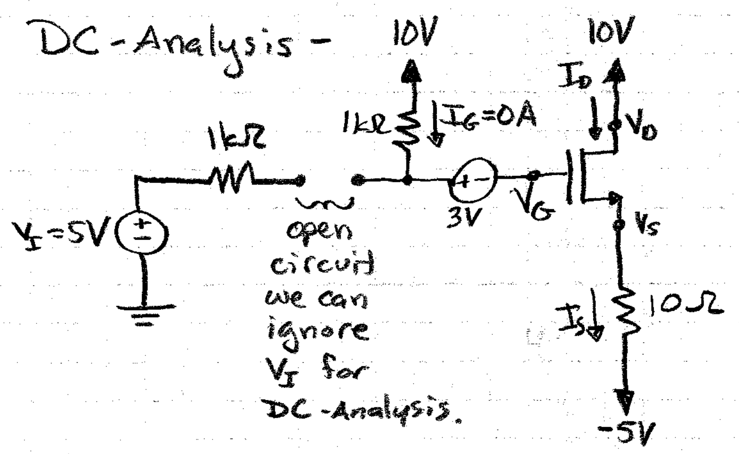
$$V_S = \frac{20 \pm \sqrt{20^2 - 4(81)}}{2} = \frac{20 \pm \sqrt{76}}{2} = \text{~~14.16 V~~ or } 5.64V$$

Not Possible — Exceeds Source Voltages.

$$\therefore V_S = 5.64V$$

5) $V_t = 1V$ $\quad K_n' W/L = 10\mu A/V^2$ $\quad \lambda = 0$

DC-Analysis -



Note: $I_G = 0A$, so $V_G = 10V - 3V = 7V$

$V_D = 10V$

$V_D > V_G - V_t \iff 10V > 7V - 1V = 6V$ (MOSFET SAT.)

$I_S = I_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2 = \frac{1}{2} 10\mu A/V^2 (6V - V_S)^2$

$I_S = \frac{V_S + 5V}{10\Omega} \Rightarrow V_S = -5V + 10\Omega I_S$

$I_S = 10\mu \frac{1}{2} (6 + +5 - 10 I_S)^2$

$200k I_S = (11 - 10 I_S)^2 \Rightarrow 100 I_S^2 - 100.22k \cdot I_S + 121 = 0$

$\Rightarrow I_S^2 - 200.22k I_S + 121 = 0$

$I_S = \frac{200.22k \pm \sqrt{(200.22k)^2 - 4(121)}}{2} =$ 0.6e-3
200e5

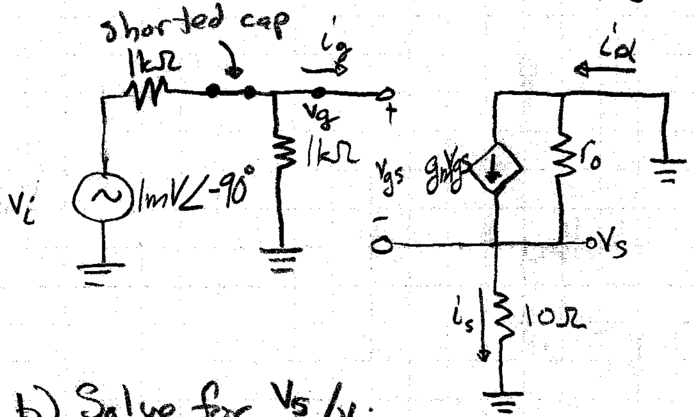
not physically possible (transistor would burn out first)

a) $I_G = 0A$, $I_D = I_S =$ 0.6mA

b) $V_G = 7V$, $V_D = 10V$, $V_S = -4.998V$

c) The Q-point is $V_{GS} = 11.998V$

6) a) Draw the small signal equivalent circuit.



$r_o = \frac{1}{\lambda I_D} = \infty$ (open)

$\Rightarrow i_d = i_s = g_m v_{gs}$

$g_m = \frac{2 I_D}{V_{GS} - V_t} = \frac{2 \times 0.18mA}{11.998V} = 30\mu A/V$

b) Solve for V_s / V_i.

$V_g = \frac{1}{2} V_i$ ← voltage divider

$V_g / V_i = 1/2$

$V_s = 10\Omega \, i_s = 10\Omega \cdot g_m v_{gs} = 10\Omega \cdot 30\mu (v_g - v_s)$

$V_s = \frac{300\mu m \, V_g}{1 + 300\mu} \approx 300\mu \, V_g$

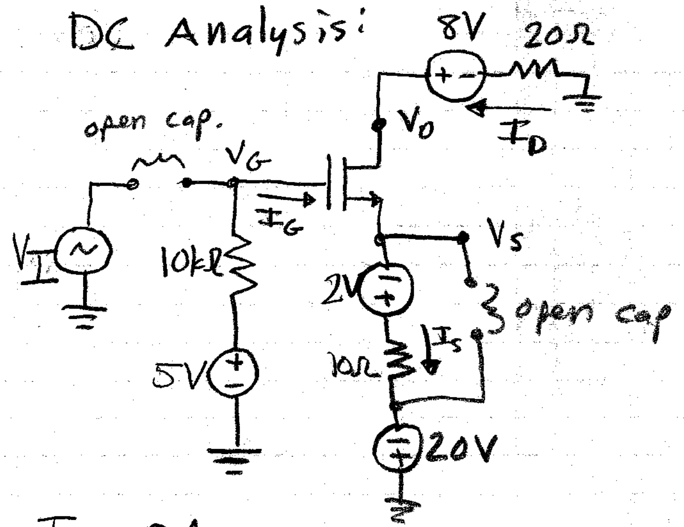
$\frac{V_s}{V_g} = 300 \, \mu V/V$

$\frac{V_g}{V_i} \cdot \frac{V_s}{V_g} = \frac{V_s}{V_i} = 300\mu V/V \cdot \frac{1}{2} = 150\mu V/V$

$\therefore V_s / V_i = 150 \mu V/V$

7) $V_t = 2V$, $K_n' W/L = 100\mu A/V^2$, $\lambda = 0$ $V_I = 3 + 1m\sin(10t)$ V

DC Analysis:



$I_G = 0A$

$V_G = 5V - (0A) \cdot 10k\Omega = 5V$

$V_D = 8V - I_D(20\Omega)$ and $I_D = \dfrac{8V - V_D}{20\Omega}$

$V_S = -20V + I_S(10\Omega) - 2V$

$I_S = I_D \Rightarrow V_S = -22V + I_D(10\Omega)$ and $I_D = \dfrac{V_S + 22V}{10\Omega}$

For now assume saturation:

$I_S = I_D = \frac{1}{2} K_n' \frac{W}{L} (V_{GS} - V_t)^2 =$

$$\frac{1}{2} 100\mu (3 - V_S)^2 = \frac{V_S + 22}{10}$$

$$\frac{1}{2}(3 - V_S)^2 = 1k\, V_S + 22k$$

$$\frac{1}{2} V_S^2 - \frac{6}{2} V_S + \frac{9}{2} - 1k V_S - 22k = 0$$

$$V_S^2 - 6V_S + 9 - 2k V_S - 44k = 0$$

$V_S = -21.7V$ or ~~2.027 kV~~ not physically possible

$V_{GS} = 5 - -21.7 = 26.7V$ $I_D = \dfrac{V_S + 22V}{10\Omega} = 30mA$

$V_{DS} = V_D - V_S = 8V - 20\Omega(I_D) - -21.7 = 29.1V$

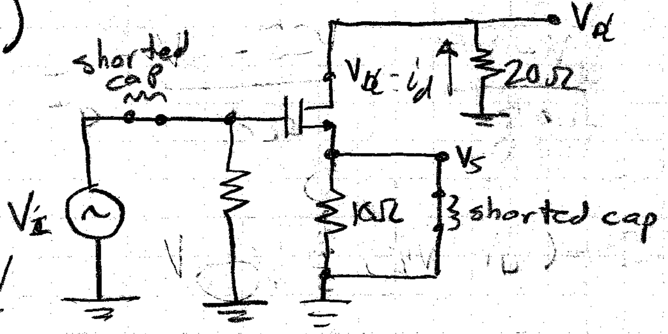
$V_{DS} - V_{GS} = 2.4 > V_t$ (FET in Saturation) ✓

ECE 2280 | HW#4 Solution

(a) $I_G = 0$, $I_D = I_S = 30mA$
(b) $V_G = 5V$ $V_S = -21.7V$ $V_D = 7.4V$
(c) $V_{GS} = 26.7V$

8)

a)



$V_I = 1m\sin(10t)$

b) From Circuit $v_s = 0V$

$$i_d = g_m v_{gs} = \frac{2 I_D v_{gs}}{V_{OV}} = \frac{2(30mA)}{24.7} v_{gs} = 2.43\,mA/V \cdot v_{gs}$$

$$v_g = v_i \Rightarrow \frac{v_g}{v_i} = 1 \qquad \frac{v_{gs}}{v_i} = \frac{v_g}{v_i} = 1$$

$$v_d = -i_d(20\Omega)$$

$$\frac{v_d}{i_d} = -20\Omega \qquad \frac{i_d}{v_{gs}} = 2.43\,mA/V \qquad \frac{v_{gs}}{v_i} = 1$$

$$\frac{v_d}{i_d} \cdot \frac{i_d}{v_{gs}} \cdot \frac{v_{gs}}{v_i} = \frac{v_d}{v_i} = -20\frac{V}{A}(2.43mA/V) \cdot 1\frac{V}{V} = -48.6\frac{mV}{V}$$

$$\therefore \frac{v_d}{v_i} = -48.6\frac{mV}{V}$$