ECE2280 **Homework #6** **Spring 2007**

1. If V_t=2, $k_n'(W/L)$=2mA/V^2, λ=0, what is the value of the current, I_D, flowing through an NMOS transistor for the following applied voltages:
   (a) $V_G=5$, $V_D=10$, $V_S=4$
   (b) $V_G=5$, $V_D=10$, $V_S=-5$
   (c) $V_G=5$, $V_D=2$, $V_S=-5$
   (d) $V_G=3$, $V_D=1$, $V_S=1$
   (e) $V_G=-5$, $V_D=0$, $V_S=-10$

(a) $V_{GS} = 5-4 = 1V < V_t \therefore$ off

$\boxed{I_D = 0}$

(b) $V_{GS} = 5-(-5) = +10 > V_t \therefore$ ON

$V_{DS} = 10-(-5) = 10+5 = 15$

$+15 = V_{DS} > (V_{GS}-V_t) = 10-2 = 8V$ SAT

$I_D = \frac{1}{2} k_n'(\frac{W}{L})(V_{GS}-V_t)^2 = \frac{1}{2}(2m)(+10-2)^2 = \boxed{64mA}$

(c) $V_{GS} = 5-(-5) = +10 > V_t \therefore$ ON

$V_{DS} = 2-(-5) = +7$

$(V_{DS}=7) < (V_{GS}-V_t) = 8 \therefore$ Triode

$I_D = k_n'(\frac{W}{L})[(V_{GS}-V_t)V_{DS} - \frac{1}{2}V_{DS}^2] = 2m[8(7)-\frac{1}{2}7^2] = \boxed{63mA}$

(d) $V_{GS} = V_G - V_S = 3-1 = 2$

$V_D = 1 = (V_G - V_t) = 3-2 = 1$ (either eq.)

SAT: $I_D = \frac{1}{2}(2m)(2-2)^2 = 0$ A. $V_{GS} = V_t$ (just on the threshold of turning on $\therefore$ $\boxed{I=0}$)

(e.) $V_{GS} = -5+(+10) = 5V$

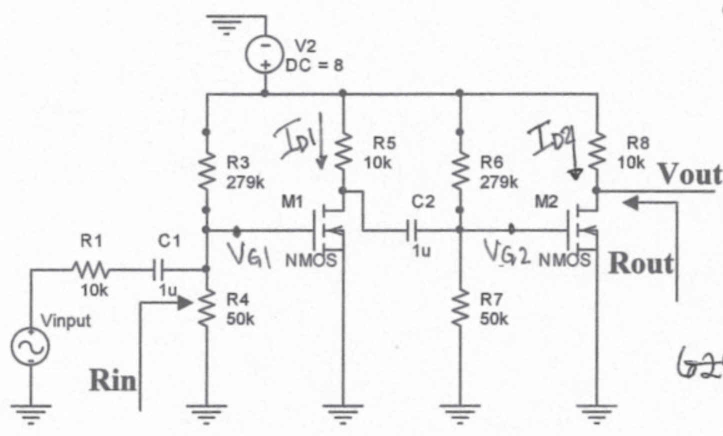
$V_{DS} = +10$

$10 = V_{DS} \geq (V_{GS}-V_t) = 5-2 = +3V$ SAT

$I_D = \frac{1}{2}(2m)(3)^2 = \boxed{9mA}$

2. (for each circuit: (a) worth 1 problem (b)-(e) worth 1 problem, (f)-(g) worth 1 problem; total=9 problems) For each circuit below, answer the following using: (i) V_t=0.8V, $k_n'(W/L)$=3.2mA/V^2, λ=0; (ii) V_t=1V, $k_n'(W/L)$=1.6mA/V^2, λ=0; (iii) V_t=1.5V, $k_n'(W/L)$=1mA/V^2, λ=0. Assume V_{input} is an AC signal.

(a) Solve the DC circuit to find I_D and V_{GS}(assume caps open) for transistor(s)
(b) Draw the small-signal equivalent circuit (assume caps shorted)
(c) Find the midband gain V_{out}/V_{input} (V_{input} is an AC source)
(d) Find Rin
(e) Find Rout
(f) Solve the circuit below by including the capacitors in the AC small-signal equivalent circuit. Solve the frequency transfer function V_{out}/V_{input} and state the low frequency value, f_L in Hz.
(g) State the upper corner frequency for the entire amplifier, f_H in Hz if C_{gs}=1pF and C_{gd}=0.1pF.



(i)

a. DC: $V_{G1} = V_{G2}$

$I_{D1} = I_{D2}$

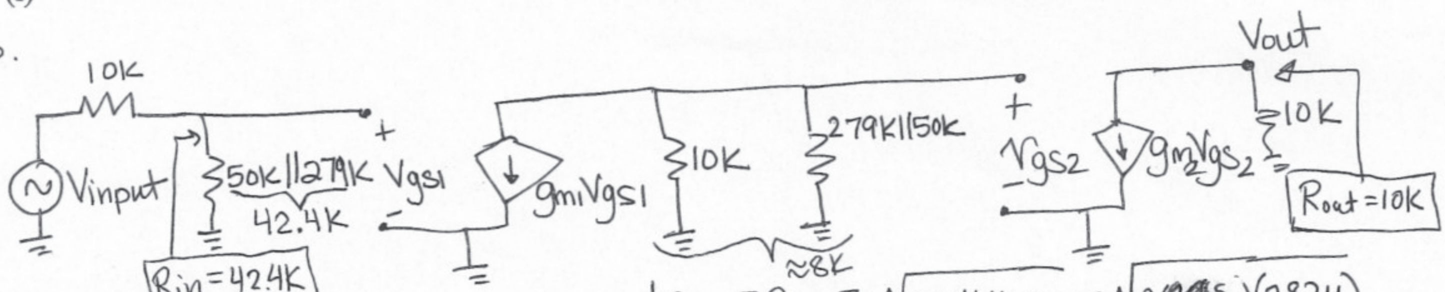
Assume SAT:

$V_{G1} = \frac{8(50K)}{50K+279K} = \boxed{1.22V = V_{GS}}$

$I_D = \frac{1}{2}(3.2m)(1.22-\overset{V_G - V_S}{0}-0.8)^2$

$\cancel{625}\ I_D = 1.6m(.42)^2 = \boxed{282\mu A}$

b.



$g_{m1} = g_{m2} = \sqrt{2k_n'(\frac{W}{L}) \cdot I_D} = \sqrt{2(3.2m)(282\mu)}$

$= \sqrt{1.805\mu}\ A/V = 1.34mA/V$

c. $V_{out} = -g_{m2} V_{gs2} \cdot 10K$

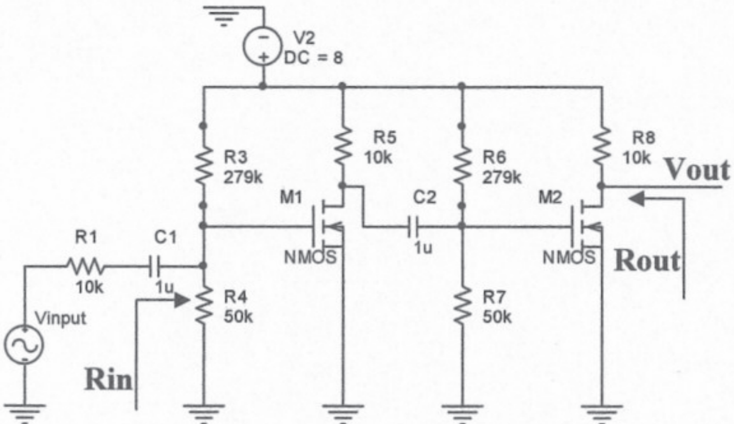
$V_{gs2} = -g_{m1} V_{gs1} (10K||279K||50K)$

$V_{gs1} = \frac{V_{in}(42.4K)}{(42.4K+10K)} = 0.81V_{in}$

$V_{gs2} = -1.34m V_{gs1}(8K) = -10.72 V_{gs1}$

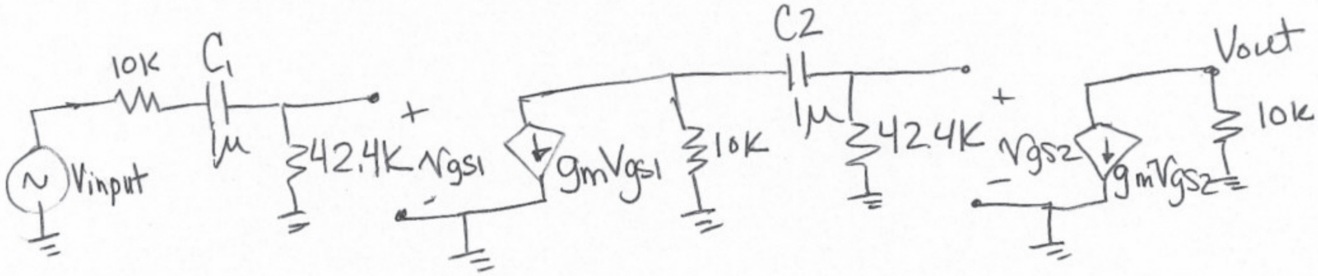
$V_{out} = -1.34m V_{in}(.81)(-10.72) \cdot 10K$

$\frac{V_{out}}{V_{in}} \boxed{+117 V/V}$



$g_m = 1.34m.$

f)



$$V_o = -g_m V_{gs2} \cdot 10K$$

$$V_{gs2} = \left[\frac{-g_m V_{gs1}(10K)}{10K + 42.4K + \frac{1}{1\mu \cdot s}}\right] 42.4K = \frac{-1.34m(10K)(1\mu s)(42.4K)V_{gs1}}{((52.4K)(1\mu \cdot s)+1)} = \frac{-13.4\mu \cdot s\, V_{gs1}\, 42.4K}{(52.4m \cdot s+1)}$$

$$V_{gs1} = \frac{V_{input}(42.4K)}{52.4K + \frac{1}{1\mu \cdot s}} = \frac{V_{input}\,(42.4K)(1\mu)\cdot s}{(52.4K(1\mu)s+1)} = \frac{42.4m\, V_{input}\cdot s}{(52.4m \cdot s+1)}$$

$$\frac{V_o}{V_{input}} = \frac{+(1.34m)(13.4\mu)s^2(42.4m)42.4K(10K)}{(52.4m \cdot s+1)^2} = \boxed{\frac{0.323 \cdot s^2}{(52.4m \cdot s+1)^2}}$$

pole value = cap * R seen by cap

Note this value is $\left[\frac{1}{C_1(R_1 + R_3 \| R_4)}\right]$ OR $\frac{1}{C_2(10K + 279K \| 50K)}$

$\frac{1}{52.4m} = 19 \frac{rad}{sec.}$

$$f_L = \frac{\left(\frac{1}{52.4m}\right)}{2\pi} = \boxed{3Hz}$$

g. $\omega_{H_1} = \frac{1}{R'_{sig}(C_{eq,1} + C_{gs,1})}$

$C_{eq_1} = C_{gd}(1 + g_m R'_L) = 0.1p(1 + 1.34m(8K))$

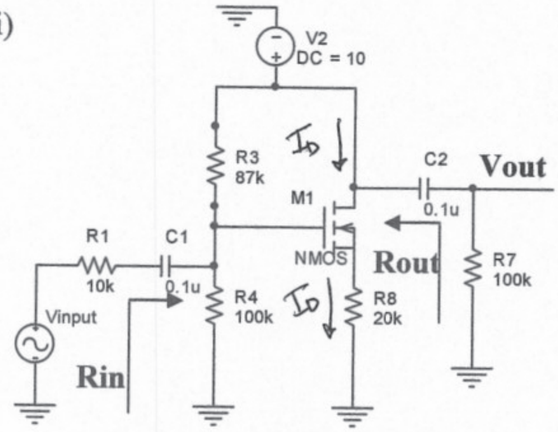
$C_{eq,1} = 1.172 \times 10^{-12}$

$$\omega_{H_1} = \frac{1}{8K(1.172 \times 10^{-12} + 1p)} = 57.6\,M\,rad/sec.$$

lower value $\rightarrow \omega_{H_2} = \frac{1}{8K[0.1p(1+1.34m(10K)) + 1p]} = 51.2\,M\,rad/sec = \boxed{8.2\,MHz}$

$V_t = 1V, \quad K_n'(\frac{W}{L}) = 1.6m\ A/V^2$

(ii)



DC: $V_G = \frac{10(100K)}{187K} = 5.35V$

a.

$V_S = I_D(20K)$

Assume SAT:

$I_D = \frac{1}{2}(1.6m)(5.35 - I_D(20K) - 1)^2$

$\downarrow$ solve

$I_D = \boxed{0.193m},\ 0.245m$

$V_S = .245m(20K) = 4.9V$

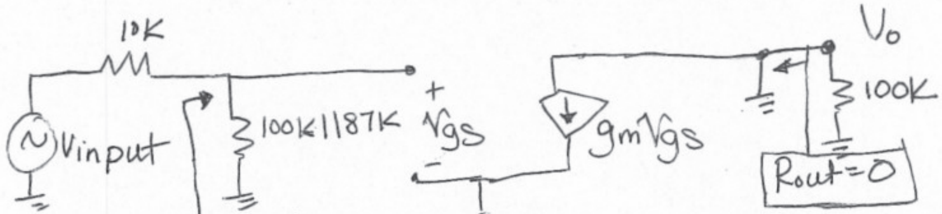
$V_{GS} = .45 < V_t \therefore$ off

$V_S = .193m(20K) = 3.86$

$V_{GS} = 5.35 - 3.86$

$\boxed{V_{GS} = 1.49} > V_t \therefore$ ON

b.



$\boxed{R_{in} \cong 47K}$

$\boxed{R_{out} = 0}$

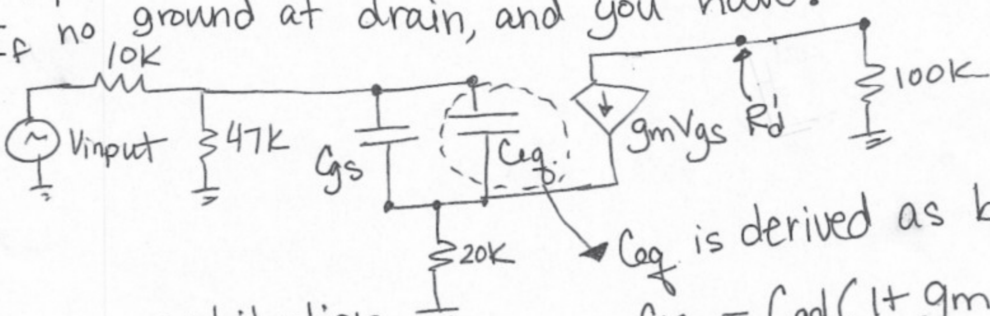
$g_m = \sqrt{2(1.6m)(0.193m)} = 0.79m$

$g_m = K_n'(\frac{W}{L})(V_{GS} - V_t) \cong 0.78m$

$V_o = 0V$ so $\boxed{\frac{V_o}{V_{input}} = 0\ V/V}$

(f) $\frac{V_o}{V_{input}} = 0\ V/V$ no change, no low or high freq. since 0V gain. (no gain) (does not ever work)

Note: → If no ground at drain, and you have:



Ceq is derived as before so

$C_{eq.} = C_{gd}(1 + g_m R_d')$ ← resistance seen at drain node

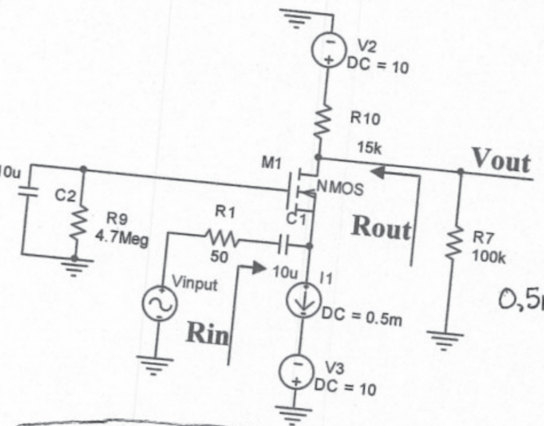
$C_{eq.} = 0.1p(1 + .8m(100K))$

frequency contribution is still : $\frac{1}{C * R_{\text{seen by cap.}}}$

$\therefore \frac{1}{R_{\text{seen by cap}}(C)}$ where $C = (C_{gs} + C_{eq.})$

$\frac{1}{[(47K \| 10K) + 20K](C_{gs} + C_{eq})} = \frac{1}{28.2K(8.1p + 1p)} = 3.9M\ \frac{rad}{sec.}$

(iii)



DC: $V_G = 0$ $K_n'(\frac{W}{L}) = 1m\,A/V^2$

$V_t = 1.5V$

$I_D = 0.5m$

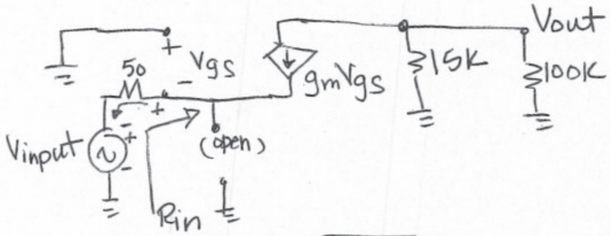
Assume SAT:

$0.5m = I_D = \frac{1}{2}(1m)(0 - V_S - 1.5)^2$

$\sqrt{1} = -V_S - 1.5$

$V_S = -1.5 \pm 1 = \underline{\underline{-2.5}}, -0.5$ ← NOT ON since $V_{GS} < 1.5$

$\therefore \boxed{V_{GS} = +2.5,\ I_D = 0.5m}$



$g_m = \sqrt{2K_n'(\frac{W}{L})I_D} = \sqrt{2(1m)(.5m)} = 1m\,A/V$

OR $g_m = K_n'(\frac{W}{L})(V_{GS} - V_t) = 1m\,(2.5 - 1.5) = 1m\,A/V$

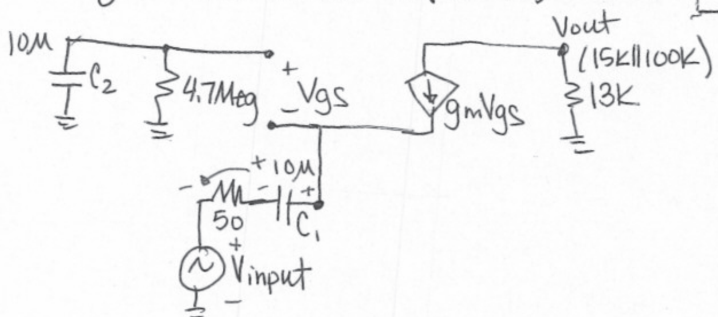
$V_{out} = -g_m V_{gs}\,\underbrace{(15K \| 100K)}_{13K}$

$V_{gs} = 0 - (g_m V_{gs}(50) + V_{input})$

$V_{gs}(1 + g_m(50)) = V_{input}$

$V_{gs} = \frac{V_{input}}{1 + g_m(50)} = 0.95\,V_{input}$

$V_{out} = -13(.95)\,V_{input} \Rightarrow \frac{V_{out}}{V_{input}} \cong \boxed{-12\,V/V}$



$V_{out} = -g_m V_{gs}(13K)$

$V_{gs} = 0 - \left(g_m V_{gs}\left(\frac{1}{C_1 s} + 50\right) + V_{input}\right)$

$V_{gs} = \frac{V_{input}}{1 + g_m\left(\frac{1}{C_1 s} + 50\right)} = \frac{V_{input}\,C_1 s}{C_1 s(1 + g_m 50) + g_m}$

$V_{gs} = \frac{V_{input}(10\mu)s}{10.5\mu \cdot s + 1m} = \frac{V_{input}(10\mu)s}{1m\left(\frac{10.5\mu s}{1m} + 1\right)}$

$V_{gs} = \frac{10m\,V_{input} \cdot s}{(10.5m \cdot s + 1)}$

$V_{out} = \frac{-13(10m)s \cdot V_{input}}{(10.5m \cdot s + 1)} \Rightarrow$

$\boxed{\frac{V_{out}}{V_{input}} = \frac{0.13s}{(10.5m \cdot s + 1)}}$

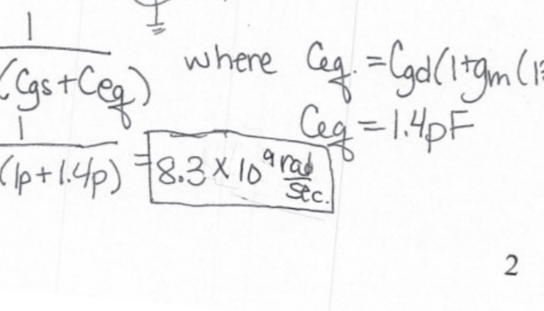
Note ⇒ $\frac{1}{C_1\left(\frac{1}{g_m} + 50\right)} = \frac{1}{10.5\mu} \approx 95$

cap * R seen by Cap

$\boxed{\begin{array}{l} f_L = 15Hz \\ f_H = 1.3 \times 10^9 Hz \end{array}}$

W_H:

$C_{gd} = 0.1pF,\ C_{gs} = 1pF$



R seen by $(C_{gs} + C_{eq})$ is $\underline{50}$ (Vinput is shorted) so

$\omega_H = \frac{1}{50(C_{gs} + C_{eq})}$ where $C_{eq} = C_{gd}(1 + g_m(13K))$

$C_{eq} = 1.4pF$

$= \frac{1}{50(1p + 1.4p)} = \boxed{8.3 \times 10^9 \frac{rad}{sec.}}$