

**Signals**

A **DC** (direct current) signal refers to a fixed voltage whose polarity never reverses. {Ex. 5V,-15V}

An **AC** (alternating current) occurs when charge carriers periodically reverse their direction of movement. {Ex. Sinusoid =>  $5\sin(10t)$ , Square Waves, Sawtooth-shaped}

- The voltage of an AC power source changes from instant to instant in time.
- Wall plug is AC with a frequency of 60 hertz and 120V
  -
- RMS value =

**Real signals** such as your voice, environmental sensors, etc. are time-varying voltages or currents that carry information.

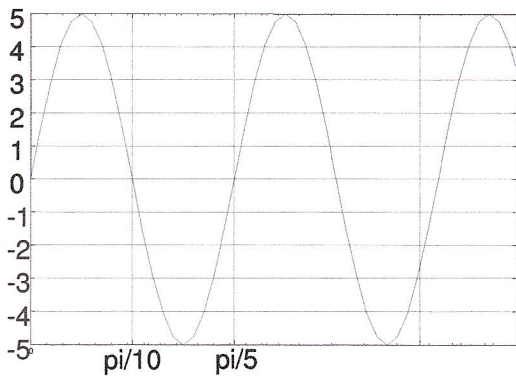
- Transducers transform one form of energy into another:
  - Ex: Microphone, Camera, Thermistor or other thermal sensor, Potentiometer, Light sensor, Computer, etc.
- **Sine waves are "pretend" signals**
  - Although sine waves are not *real* signals, we use them to simulate signals all the time, both in calculations and in the lab. This makes sense because all signals can be thought of as being made up of a spectrum of sine waves.

These types of signals can be hard to characterize mathematically. If a signal is periodic but arbitrary in amplitude, recall that it can be expressed by the Fourier series(a series of sinewaves of different frequencies and amplitudes).

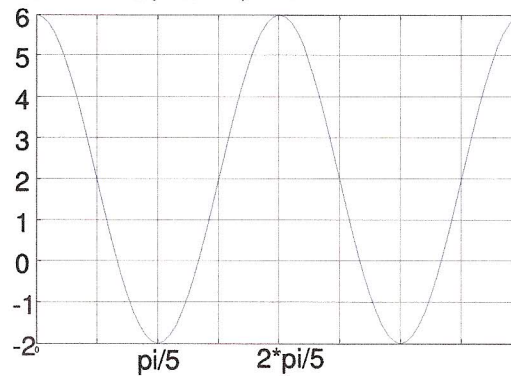
**Example #5**

Sketch the following waveforms. Identify the dc component of the waveform and the ac component of the waveform.

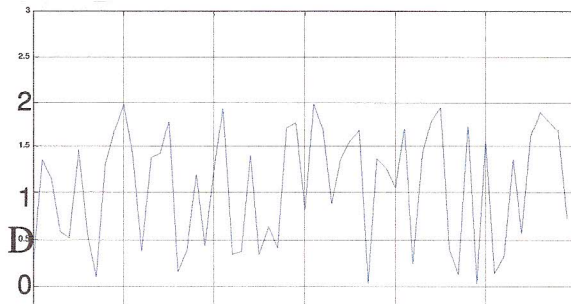
a.  $V_s =$



b.  $V_s =$

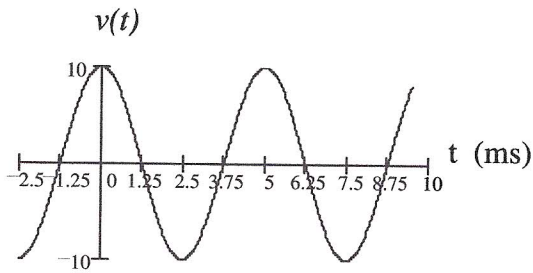


c.  $V_s =$

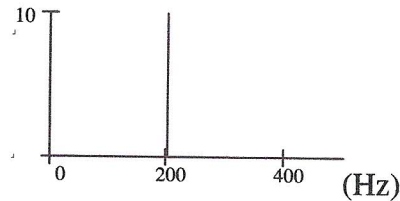


Sine wave:

Time domain:



Frequency domain:



amplitude :=

$$V_{RMS} :=$$

$$f = 200 \cdot \text{Hz}$$

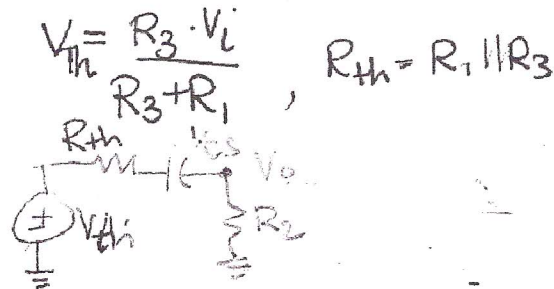
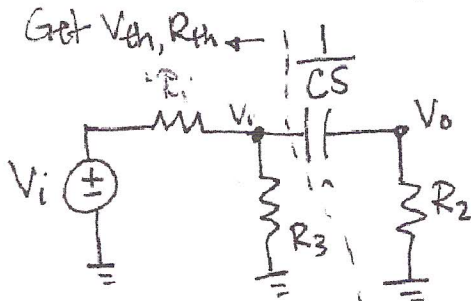
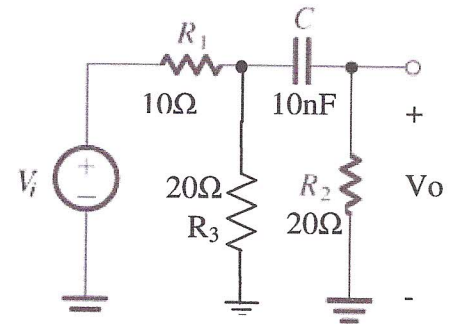
$$T := 5 \cdot \text{ms}$$

$$f := \frac{1}{T}$$

$$\omega := 2 \cdot \pi \cdot f$$

**Example #6**

When analyzing a time dependent element (capacitors), translate into frequency domain  $\Rightarrow C = \frac{1}{j\omega C} = \frac{1}{sC}$  and then analyze the circuit using normal circuit analysis techniques. Analyze the circuit to the right to find the transfer function  $\frac{V_o}{V_i}$ . Solve the circuit symbolically first (with  $R_1, R_2, R_3, C$ ) and then plug in their values.



$$\frac{V_o}{V_i} = \frac{R_2 C \cdot s \cdot R_3 \cdot V_1}{(R_1 + R_3)([R_2 + R_1 || R_3] \cdot Cs + 1)} = \frac{20(10n) s (20)}{(10 + 20)(266.7n \cdot 10n \cdot s + 1)}$$

$$\frac{V_o}{V_i} = \frac{4 \times 10^{-6} \cdot s}{30(266.7n \cdot s + 1)} = \frac{(133n) \cdot s}{(266.7n \cdot s + 1)}$$

What does this equation mean? By substituting  $s=j\omega$  in the above equation. The magnitude of the equation is:

$$\left| \frac{133n(\omega)j}{(266.7n(\omega)j + 1)} \right| = \frac{|133n(\omega)j|}{|(266.7n(\omega)j + 1)|} = \frac{133n(\omega)}{\sqrt{(266.7n(\omega))^2 + 1^2}}$$

This magnitude can now be graphed with the x-axis as  $\omega$  and the y axis as the calculated value. This is one of the graphs used for the Bode plots. To plot this, an understanding of dB is needed.

**Decibels**

Your ears respond to sound logarithmically, both in frequency and in intensity.

Musical octaves are in ratios of two. "A" in the middle octave is 220 Hz, in the next, 440 Hz, then 880 Hz, etc...

It takes about ten times as much power for you to sense one sound as twice as loud as another.

$$10x \text{ power} \approx 2x \text{ loudness}$$

A bel is such a 10x ratio of power. Power ratio expressed in bels =  $\log\left(\frac{P_2}{P_1}\right)$  bels. The bel is named for Alexander Graham Bell.

It is a logarithmic expression of a unitless ratio (like gain).

The bel unit is never actually used, instead we use the decibel (dB, 1/10<sup>th</sup> of a bel).

$$\text{Power ratio expressed in dB} = 10 \cdot \log\left(\frac{P_2}{P_1}\right) \text{ dB}$$

dB are also used to express voltage and current ratios, which related to power when squared.  $P = \frac{V^2}{R} = I^2 \cdot R$

$$\text{Voltage ratio expressed in dB} = 10 \cdot \log\left(\frac{V_2^2}{V_1^2}\right) \text{ dB} = 20 \cdot \log\left(\frac{V_2}{V_1}\right) \text{ dB}$$

$$\text{Current ratio expressed in dB} = 20 \cdot \log\left(\frac{I_2}{I_1}\right) \text{ dB}$$

These are the most common formulas used for dB

Some common ratios expressed as dB

$20 \cdot \log\left(\frac{1}{\sqrt{2}}\right) = -3.01 \text{ dB}$	$10^{\frac{3}{20}} = 0.708$	$20 \cdot \log(\sqrt{2}) = 3.01 \text{ dB}$	$10^{\frac{3 \text{ dB}}{20}} = 1.413$
$20 \cdot \log\left(\frac{1}{2}\right) = -6.021 \text{ dB}$	$10^{\frac{6}{20}} = 0.501$	$20 \cdot \log(2) = 6.021 \text{ dB}$	$10^{\frac{6 \text{ dB}}{20}} = 1.995$
$20 \cdot \log\left(\frac{1}{10}\right) = -20 \text{ dB}$	$10^{\frac{20}{20}} = 0.1$	$20 \cdot \log(10) = 20 \text{ dB}$	$10^{\frac{20 \text{ dB}}{20}} = 10$
$20 \cdot \log\left(\frac{1}{100}\right) = -40 \text{ dB}$	$10^{\frac{40}{20}} = 0.01$	$20 \cdot \log(100) = 40 \text{ dB}$	$10^{\frac{40 \text{ dB}}{20}} = 100$

We will use dB fairly commonly in this class, especially when talking about frequency response curves.

**Example #7**

The frequency domain expression for the output over the input of a circuit is solved to be

$$\frac{\text{output}}{\text{input}} = \frac{10^5 (s + 5)}{(s + 1)(s + 5000)}$$

Substitute  $s=j\omega$  into the above equation and calculate the magnitude(dB) and

phase (degrees). Plug in values for  $\omega$  equal to  $10^{-1}$ , 0.8, 0.9,  $10^0$ , 2, 3, 4, 5, 6, 7,  $10^1$ ,  $10^2$ ,  $10^3$ , 3000, 4000, 5000, 6000, 7000,  $10^4$ ,  $10^5$  rad/sec and plot these values on a semilog graph for both magnitude and phase. Recall that magnitude,  $|a+bj| = \sqrt{a^2 + b^2}$  and the phase,  $\angle(a + bj) = \tan^{-1}(b/a)$

$$\text{Magnitude: } \left| \frac{\text{output}}{\text{input}} \right| = \left| \frac{10^5 (j\omega + 5)}{(j\omega + 1)(j\omega + 5000)} \right| = \frac{|10^5| |(j\omega + 5)|}{|(j\omega + 1)| |(j\omega + 5000)|} = \frac{\sqrt{(10^5)^2 + 0^2} \sqrt{5^2 + \omega^2}}{(\sqrt{1^2 + \omega^2}) * (\sqrt{5000^2 + \omega^2})}$$

$$\text{Phase: } \angle\left(\frac{\text{output}}{\text{input}}\right) = \angle\left(\frac{10^5 (j\omega + 5)}{(j\omega + 1)(j\omega + 5000)}\right) = \frac{\angle 10^5 * \angle(j\omega + 5)}{\angle(j\omega + 1) * \angle(j\omega + 5000)} =$$

=

$$\text{magnitude} = 99.5V/V = 20 * \log(99.5V/V) = 39.96 \text{ dB}; \text{ phase} = 0 + 1.15 - 5.7 - 0.001 = -4.6 \text{ degrees}$$

