DC Review

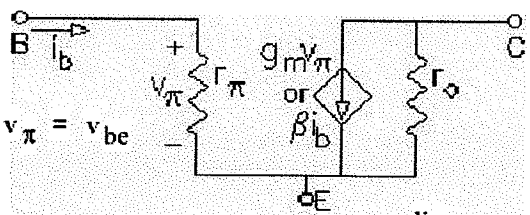
1. Assume Active
2. Label $|V_{BE}| = 0.7$
3. Take a loop through base & emitter.
4. 1 eq. with 1 unknown (I_E OR I_B)
5. Use $I_c = \alpha I_E = \beta I_B$ $\left[\alpha = \frac{\beta}{\beta+1}\right]$
6. Then you can get all Voltages
7. Check active cond. $\Rightarrow$ npn $V_C \geq V_B$

   pnp $V_C \leq V_B$

Small-signal equivalent circuit models

Same concept as that of the MOSFET.

- **Hybrid-π** model is used for the BJT:

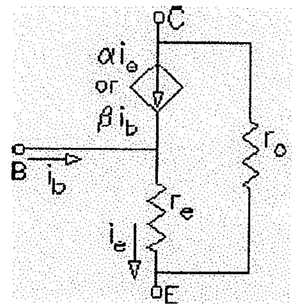


$$r_\pi = (\beta + 1)\cdot r_e \qquad i_b = \frac{v_\pi}{r_\pi}$$

$$\beta \cdot i_b = \beta \cdot \frac{v_\pi}{r_\pi} = \beta \cdot \frac{v_\pi}{(\beta+1)\cdot r_e}$$

$$g_m = \frac{\beta}{(\beta+1)\cdot r_e} = \frac{\alpha}{r_e} \simeq \frac{1}{r_e} = \text{transconductance}$$

- T-model – uses r_e instead of r_π

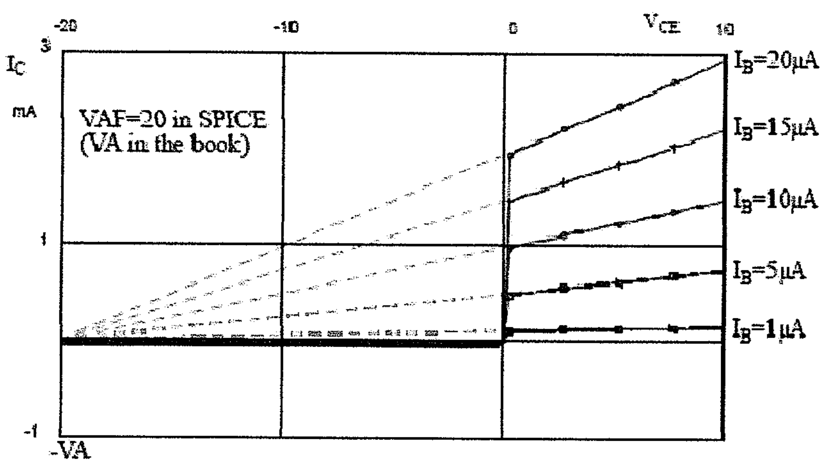


This is equivalent to Hybrid-π:

$r_e(1+\beta) = r_\pi$

Early Voltage

- The I_C vs. V_{CE} curves in the active region have a finite slope to them due to this i_C dependence on V_{CB}
- *Early* showed that these slopes all converge to one negative voltage point



The actual equation:

$$i_c = I_s e^{\frac{v_{be}}{V_T}}\left(1 + \frac{v_{ce}}{V_A}\right)$$

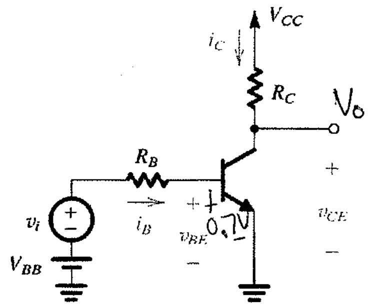
This means that the output resistance between the collector and emitter is not infinite!

Method for analyzing transistor amplifier circuits:

1). Determine dc operating point, specifically I_C
(Set ac sources to 0!!)
Note: Use method for analyzing BJT circuits at DC
2). Calculate small-signal parameters: g_m, r_π, and/or r_e
3). Set dc sources to 0
4). Replace the transistor with one of the equivalent small-signal models
5). Analyze the circuit as usual → linear circuit analysis

Example
Circuit:
β=100, V_{BB}=3V, R_C=3k
R_B=100k, V_{CC}=10V

Find the voltage gain, v_o/v_i



1). DC analysis: set v_i to 0 Assume V_{BE} = 0.7V Assume active
Redraw circuit with just dc part:

$+3 - I_B(100k) - 0.7 = 0$

$I_B = \frac{2.3}{100k} = 0.023mA$

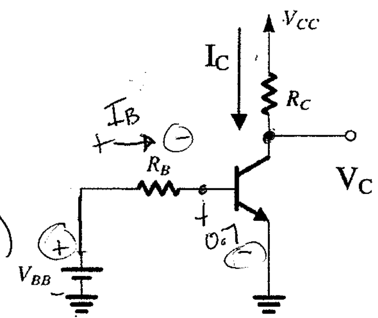
$I_C = \beta I_B = 2.3m$

$V_C = V_{CC} - I_C R_C = 10V - 2.3m(3k)$

$V_C = 3.1V \Rightarrow V_C \geq V_B$

$V_B = 0.7V$

∴ ACTIVE

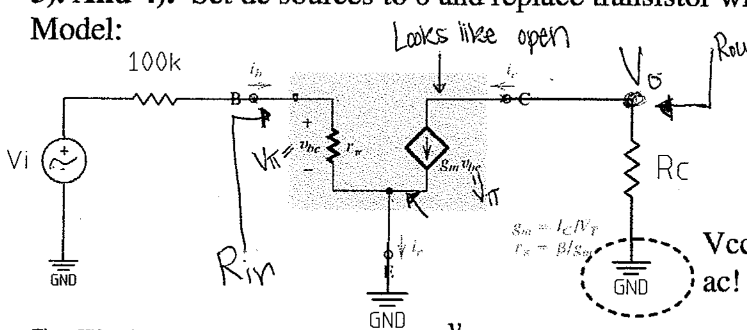


bias pt. $\Rightarrow I_C = 2.3mA$

2). Calculate small-signal parameters:

$$g_m = \frac{I_C}{V_T} = \frac{2.3}{25} = 92mA/V \qquad r_e = \frac{V_T}{I_E} = \frac{25}{(2.3/0.99)} = 10.8\Omega \qquad r_\pi = \frac{\beta}{g_m} = \frac{100}{92} = 1.09k\Omega = \frac{V_T}{I_B}$$

3). And 4). Set dc sources to 0 and replace transistor with equivalent model
Model:



Vcc grounded for ac!

AC → NOT DC!

$V_o = -g_m V_{be} \cdot R_C$

$V_{be} = \frac{V_i(r_\pi)}{r_\pi + 100k}$

5). Find requested gain: $v_o/v_i = [\frac{v_c}{v_i}]$

$\frac{V_o}{V_i} = -g_m \frac{r_\pi}{r_\pi + 100k} \cdot R_C = -92m\frac{A}{V} \cdot \frac{1.09k}{101.9k} \cdot 3k$

$\frac{V_o}{V_i} = -3.04 \, V/V$

$R_{in} = r_\pi$

$R_{out} = R_C$

BJT basic amplifier:

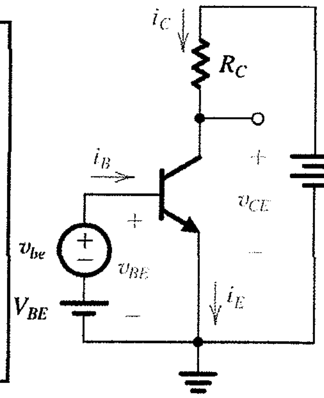
DC:

$$I_C = I_s e^{V_{BE}/V_T}$$

$$I_E = \frac{I_C}{\alpha}$$

$$I_B = \frac{I_C}{\beta}$$

$$V_C = V_{CE} = V_{CC} - I_C R_C$$



(a)

Total (instantaneous -> DC and AC)

$$i_c = I_s e^{v_{BE}/V_T}$$

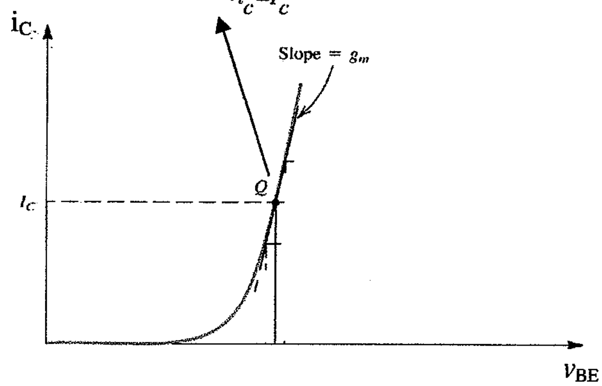
DC + AC

$$I_c + i_c = I_s e^{(V_{BE}+v_{be})/V_T} = \underbrace{I_s e^{V_{BE}/V_T}}_{I_c} e^{v_{be}/V_T}$$

$$= I_c e^{v_{be}/V_T} = I_c\left(1+\frac{v_{be}}{V_T}\right) = \underbrace{I_c}_{DC} + \underbrace{\boxed{\frac{I_c}{V_T}v_{be}}}_{AC}$$

expand by $\left[e^x \approx 1 + x\right]$ for x<<1 (V_{be}<<V_T)

$$g_m = \left.\frac{\partial i_c}{\partial v_{BE}}\right|_{i_c = I_c}$$



Look at signal component only:

$$i_c = \frac{I_c}{V_T} v_{be}$$

$$\boxed{g_m = \frac{I_c}{V_T}}$$

Transconductance
Dynamic forward resistance of BE junction

Input Resistance:

a. Input resistance looking in to base (in terms of i_b) – from base to emitter

From above: $i_C = I_c + i_c = I_c + \frac{I_c}{V_T} v_{be}$

$$i_B = \frac{i_C}{\beta} = \frac{I_c}{\beta} + \frac{I_c}{\beta V_T} v_{be}$$

$$i_B = I_B + i_b$$

$$\text{(AC)}\; i_b = \frac{I_c}{\beta V_T} v_{be} = \frac{I_B}{V_T} v_{be} \quad \frac{V}{R}$$

$$\boxed{r_\pi = \frac{V_T}{I_B}}$$

if β large($\alpha \approx 1$): good approximation is $r_e \approx \frac{1}{g_m}$

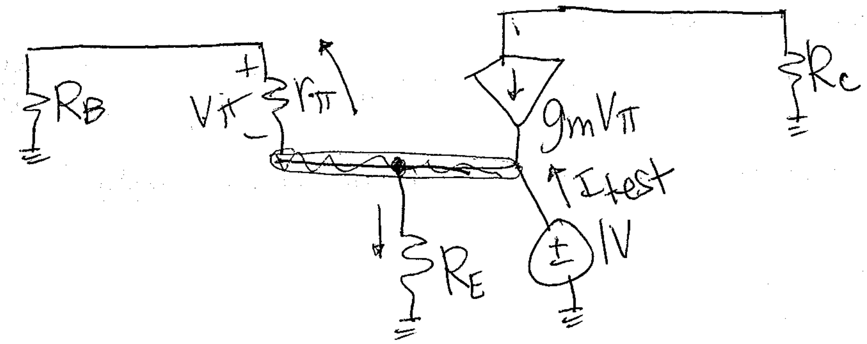
$$\boxed{r_\pi = (\beta + 1) r_e}$$

Summary of ac parameters:

thermal at room T ≈ 25mV

$g_m = \frac{I_c}{V_T}$ (I_c: DC, V_T: thermal V) $r_\pi = \frac{V_T}{I_B} = \frac{\beta}{g_m}$ (I_B: DC) $r_o \cong \frac{V_A}{I_c}$ (I_c: DC) $r_e = \frac{V_T}{I_E}$	$i_c = g_m v_{be}$ $\frac{v_c}{v_{be}} = -g_m R_c$

Resistance Reflection rule



$$R_{th} = \frac{1}{I_{test}}$$

$$-I_{test} + \frac{1}{R_E} - g_m V_\pi + \frac{1}{r_\pi + R_B} = 0$$

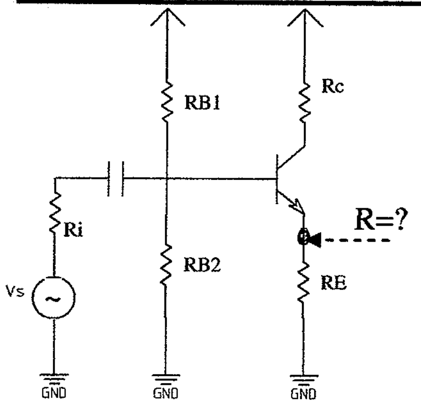
$$V_\pi = -\frac{1}{(r_\pi + R_B)} \cdot r_\pi$$

$$I_{test} = \frac{1}{R_E} + \frac{\beta}{\cancel{r_\pi}}\left(\frac{+\cancel{r_\pi}}{r_\pi + R_B}\right) + \frac{1}{r_\pi + R_B} = 0$$

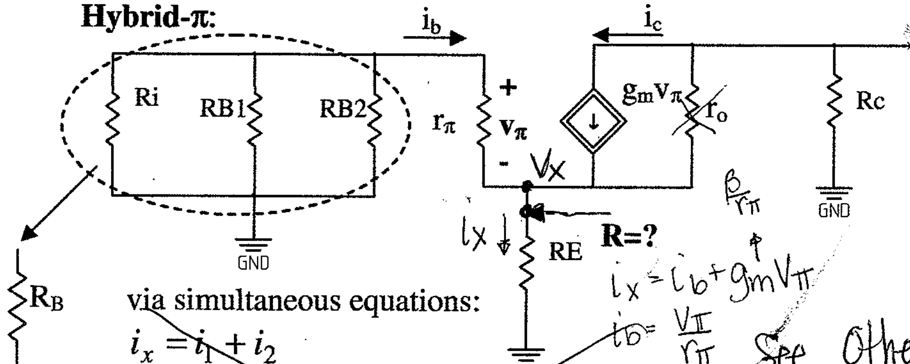
$$I_{test} = \frac{1}{R_E} + (\beta+1)\left[\frac{1}{r_\pi + R_B}\right]$$

$$R_{th} = \frac{1}{I_{test}} = \frac{1}{\frac{1}{R_E} + \frac{(\beta+1)}{r_\pi + R_B}} = R_E \parallel \frac{(r_\pi + R_B)}{(\beta+1)}$$

Resistance-Reflection Rule Between Base and Emitter:



Hybrid-π:



$i_x = i_b + g_m v_\pi$

$i_b = \frac{v_\pi}{r_\pi}$

See other page

via simultaneous equations:

$i_x = \cancel{i_1 + i_2}$

$i_1 = \frac{v_x}{R_E}$, $\quad i_2 = -i_e = -(i_b + i_c) = -i_b(\beta+1)$

$v_\pi = i_b r_\pi$, $\quad i_c = g_m v_\pi$

etc.

⋮

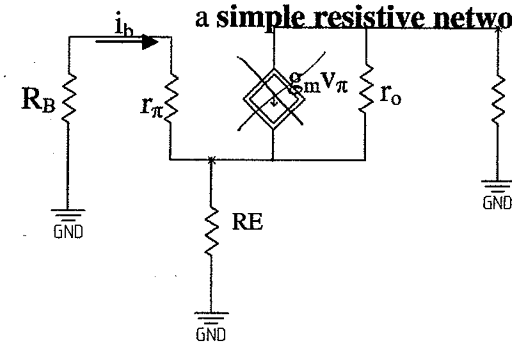
$i_x = \frac{1}{r_\pi}(v_\pi + \beta v_\pi)$

$i_x = \frac{v_\pi}{r_\pi}(1+\beta)$

$R_x = R_E \,||\, \frac{r_\pi + R_B}{(\beta+1)} \,||\, (r_o + R_c) \approx R_E \,||\, (r_e + \frac{R_B}{(\beta+1)})$

1. Eliminate VCCS
2. Scale all resistors with i_b through Thevenin by $\frac{1}{(\beta+1)}$ and

a simple resistive network remains



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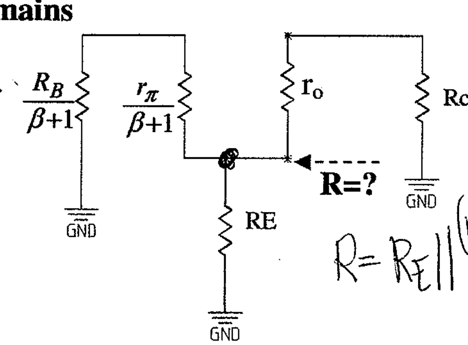
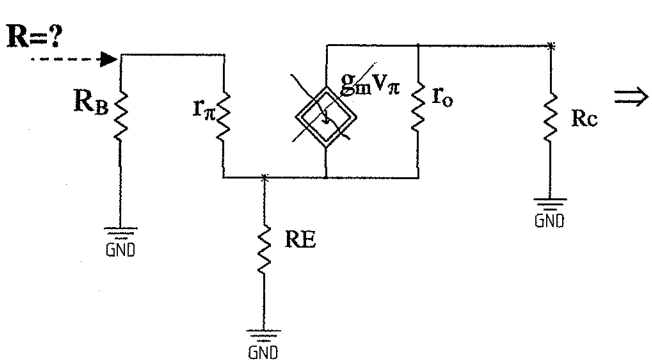


$R = R_E \,||\, \frac{(r_\pi + R_B)}{\beta+1} \,||\, (r_o + R_c)$

$R = R_B \,||\, \left[r_\pi + R_E(\beta+1) \,||\, (r_o + R_c)(\beta+1) \right]$

Same problem, but look into base instead: **R=?**



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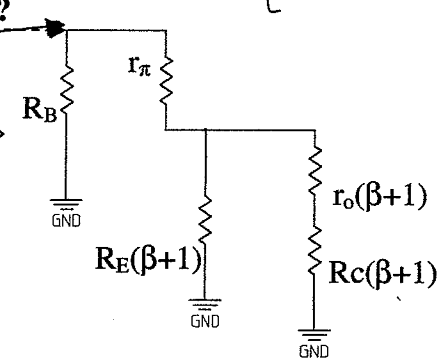
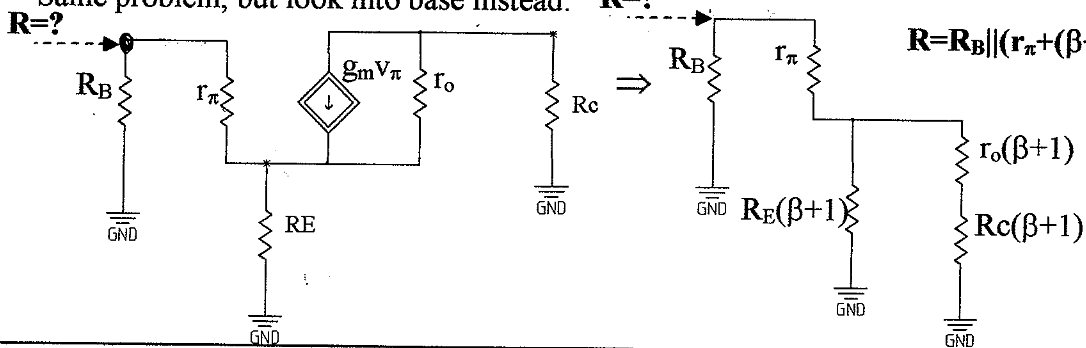


Same problem, but look into base instead: R=?



$R=R_B||(r_\pi+(\beta+1)(R_E||(r_o+Rc)))$

Summary of Resistance-Reflection Rule between base and emitter:

Applies only when you want to reflect a resistor from emitter to base or base to emitter circuit

Review of rule:

1). Temporarily remove dependent source βi_b or $g_m v_{be}$

looking above r_π

2). When looking into base: Replace resistors on emitter side with (r_π is not an Emitter R)

"R" x (β + 1) or

When looking into emitter: Replace resistors on base side with "R" / (β + 1) below r_π (includes r_π)

3). Treat circuit as a resistive network and find equivalent resistance

This works because $i_b = \dfrac{i_e}{\beta+1}$

In a nutshell: To reflect a resistor from:

E→ B multiply by (β + 1)

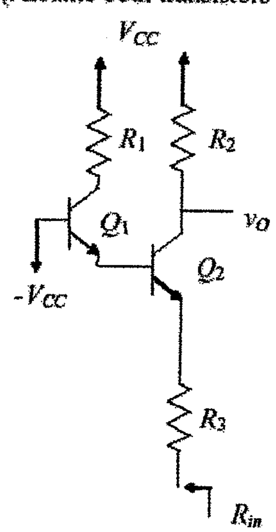
B→E divide by (β + 1)

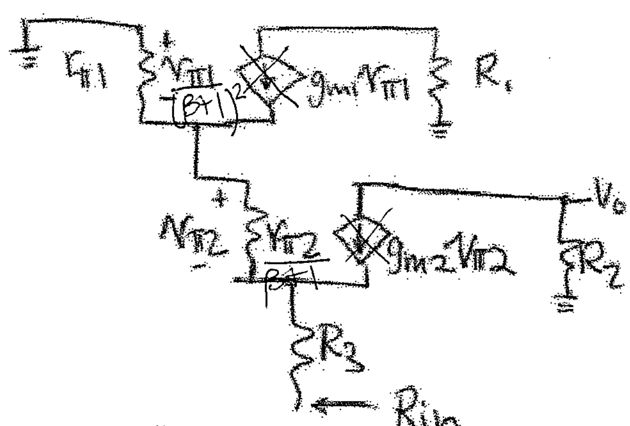
Things to keep in mind:

- Rule does NOT work for impedance looking into colle[ctor] – it is a reflection rule between base and emitter
- It works because $i_b = \dfrac{i_e}{\beta+1}$ which is a relationship betw[een] the base and emitter current!
- Finding R_{in} or R_{out} → this is just finding Thevenin equivalent resistance, R_{Th}
  - Possible methods now that you can use:
    1). Using the resistance Reflection Rule
    2). Using Thevenin equivalent methods– use these to double check homework, but on exam will not likely have time
- R_{in} or R_{out} is always between a node and ground – follo[w] all paths to ground from that node
- Applying the Reflection Rule is like turning off depend[ent] sources and multiplying resistances by (β + 1) or dividi[ng] resistance by (β + 1) and treating circuit as just a resisti[ve] network → Note that this only works because the dependent source is being accounted for through the (β [+] 1) factor!

Example: Assume the transistors below have a finite β and an infinite Early voltage.

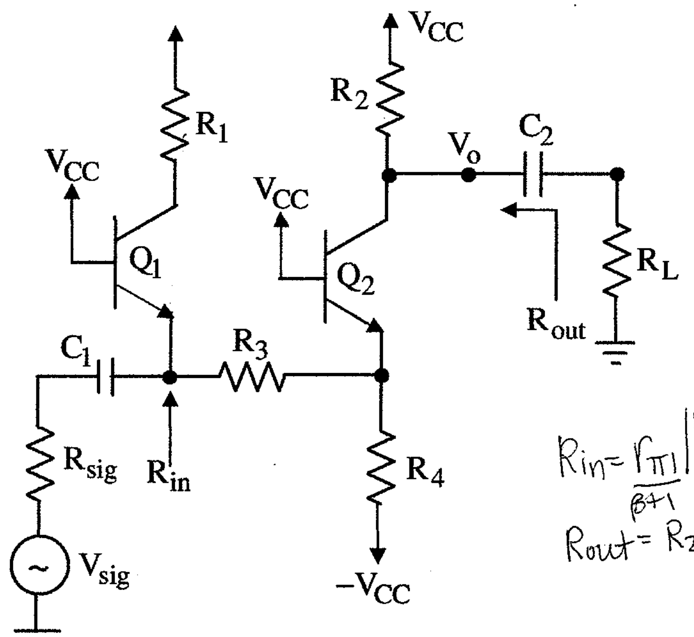
- Write an expression for the input resistance R_{in} in the circuit shown below. Your expression should include *only* real resistances (R_1, R_2, R_3, or a subset of these) and possibly β, r_{e1} or $r_{\pi1}$, and r_{e2} or $r_{\pi2}$. (Assume both transistors have the same β.) Circle your answer. *Hint: Use Resistance-Reflection rule*





$$\boxed{R_{in} = R_3 + \frac{r_{\pi2}}{(\beta+1)} + \frac{r_{\pi1}}{(\beta+1)^2}}$$

Common-Base

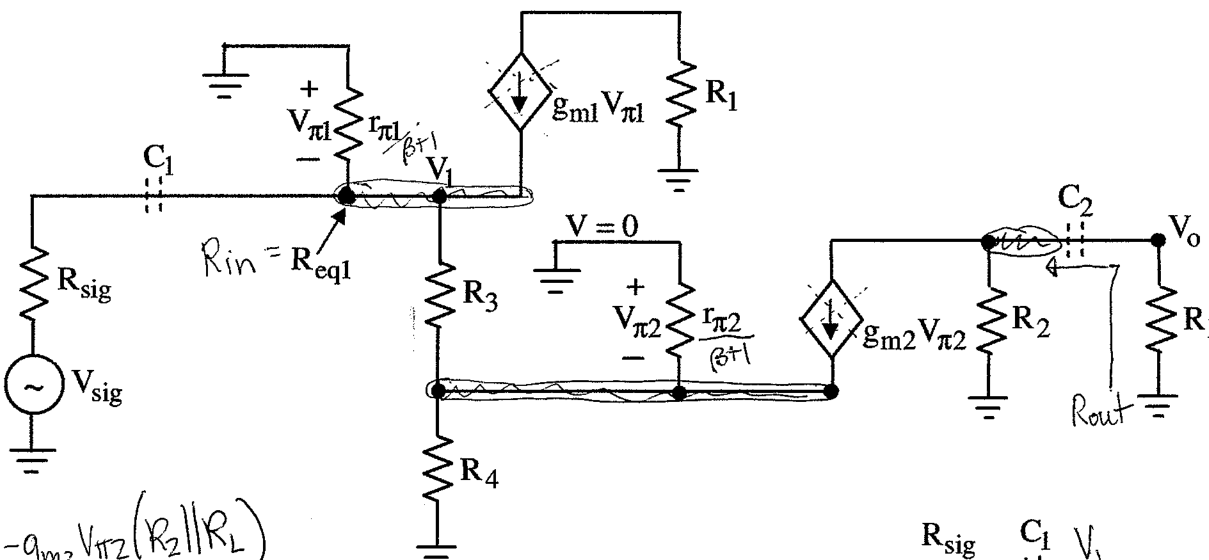


$C_1 = 10\,\text{pF}, \quad C_2 = 10\,\text{nF}, \quad \beta = 100$

Ignore r_o

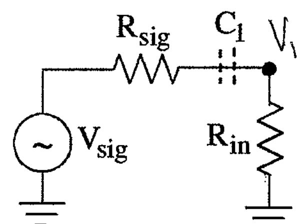
$R_{in} = \frac{r_{\pi1}}{\beta+1} \,\|\, \left[R_3 + R_4 \,\|\, \frac{r_{\pi2}}{\beta+1} \right]$

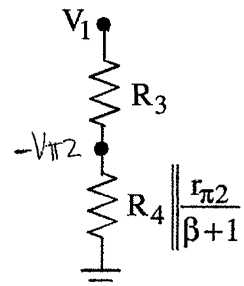
$R_{out} = R_2$



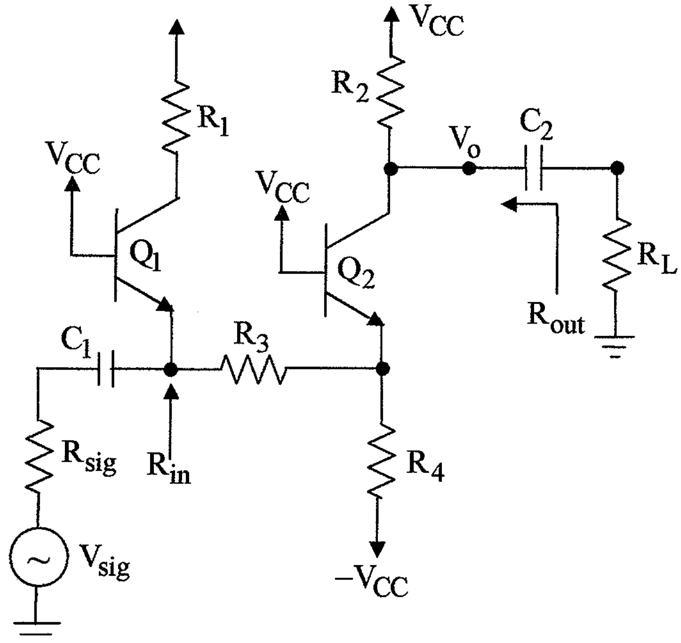
$V_o = -g_{m2} V_{\pi2} (R_2 \| R_L)$

$V_{\pi2} = \frac{-V_1 \left(R_4 \| \frac{r_{\pi2}}{\beta+1}\right)}{\left(R_4 \| \frac{r_{\pi2}}{\beta+1}\right) + R_3}, \quad V_1 = \frac{V_{sig}(R_{in})}{R_{in} + R_{sig}}$





Common-Base

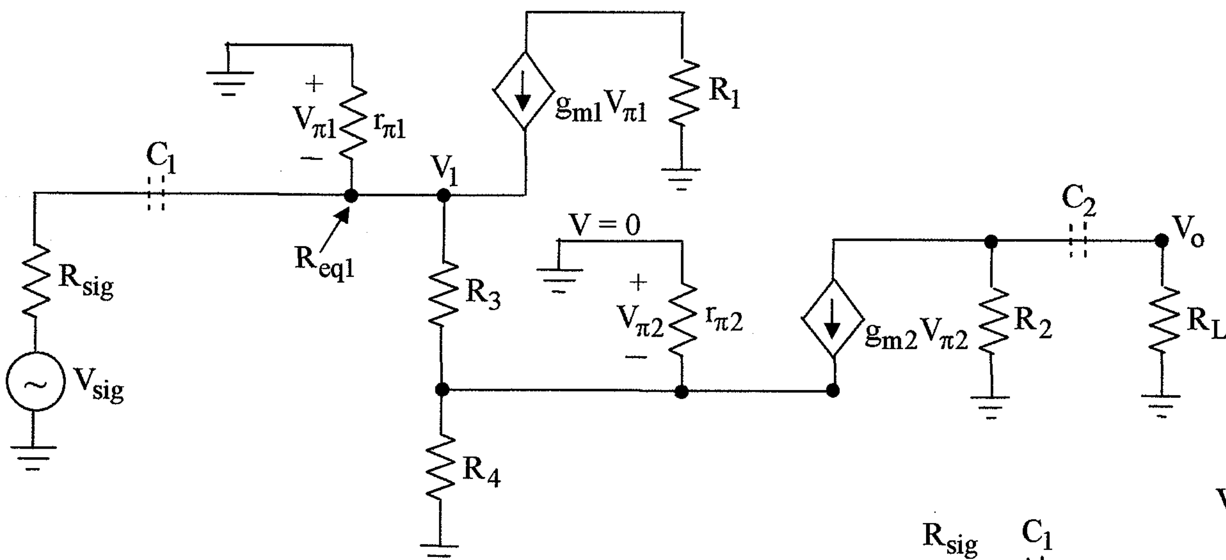


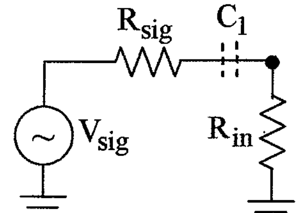
$C_1 = 10\,\text{pF}, \quad C_2 = 10\,\text{nF}, \quad \beta = 100$

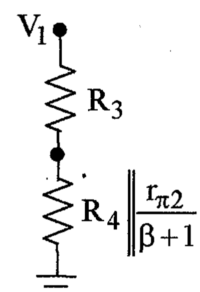
Ignore r_o

$$\boxed{R_{out} = R_2}$$

$$\boxed{R_{in} = \frac{r_{\pi 1}}{\beta+1} \Bigg\| \left(R_3 + R_4 \Bigg\| \frac{r_{\pi 2}}{\beta+1}\right)}$$





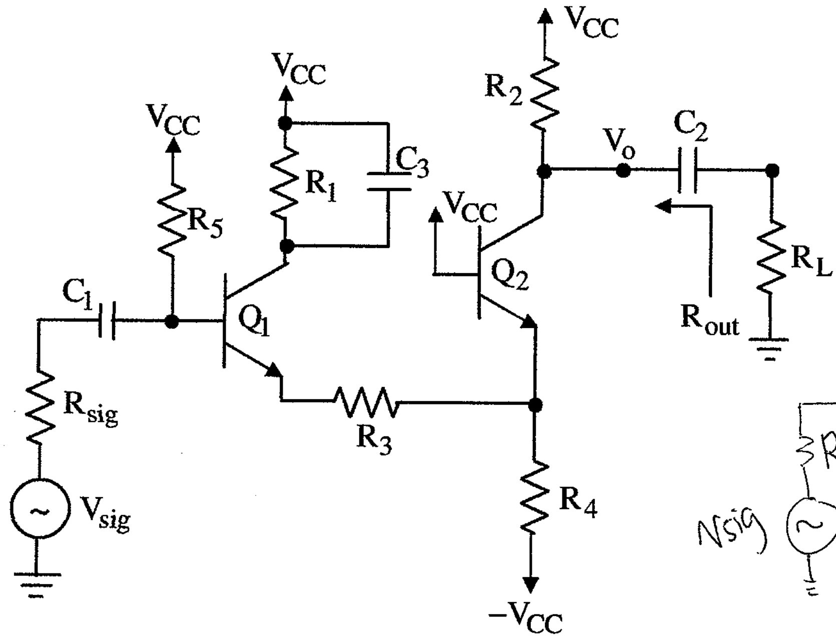


$$V_o = -g_{m2} V_{\pi 2} \left(R_2 \| R_L\right)$$

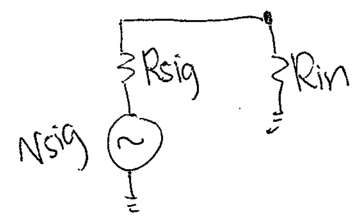
$$V_{\pi 2} = \frac{-V_1\left(R_4 \Big\| \frac{r_{\pi 2}}{\beta+1}\right)}{\left(R_4 \Big\| \frac{r_{\pi 2}}{\beta+1}\right) + R_3} \qquad V_1 = \frac{V_{sig}\left(R_{in}\right)}{R_{in} + R_{sig}}$$

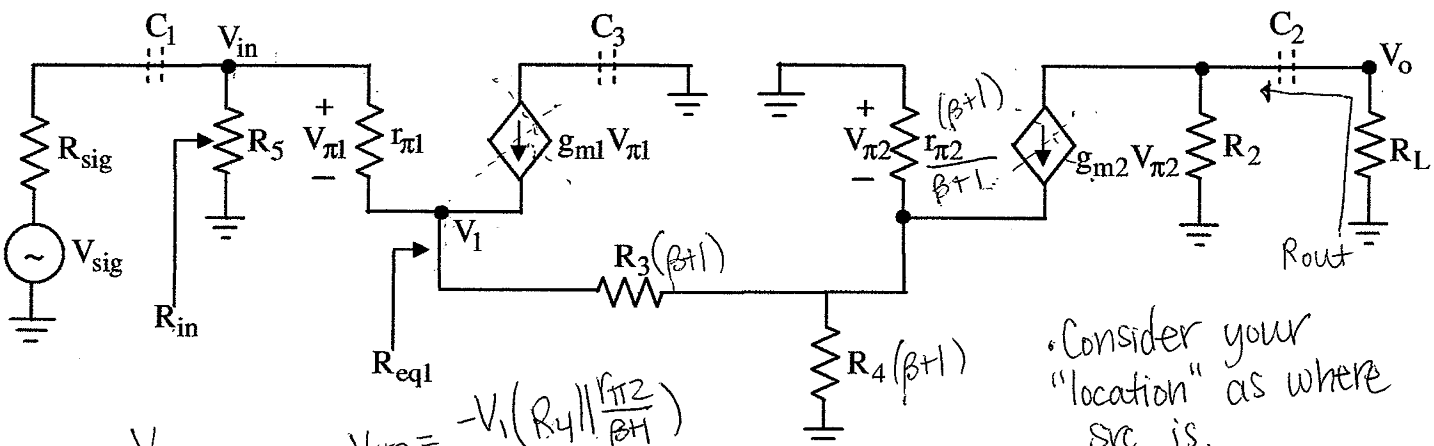
$$\boxed{\frac{V_o}{V_{sig}} = \frac{g_{m2}\left(R_2 \| R_L\right)\left(R_4 \Big\| \frac{r_{\pi 2}}{\beta+1}\right) R_{in}}{\left[\left(R_4 \Big\| \frac{r_{\pi 2}}{\beta+1}\right) + R_3\right]\left[R_{in} + R_{sig}\right]}}$$

2 Stage ⇒ Common Collector/Common Base



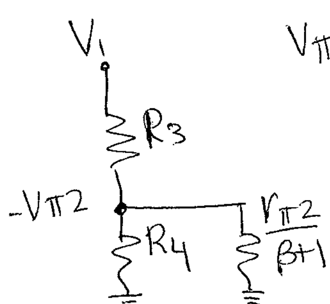
$$V_{in} = \frac{V_{sig}(R_{in})}{R_{in}+R_{sig}}$$



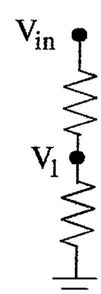


• Consider your "location" as where src is.

$$V_{\pi 2} = \frac{-V_1\left(R_4 \| \frac{r_{\pi 2}}{\beta+1}\right)}{R_3 + \left(R_4 \| \frac{r_{\pi 2}}{\beta+1}\right)}$$

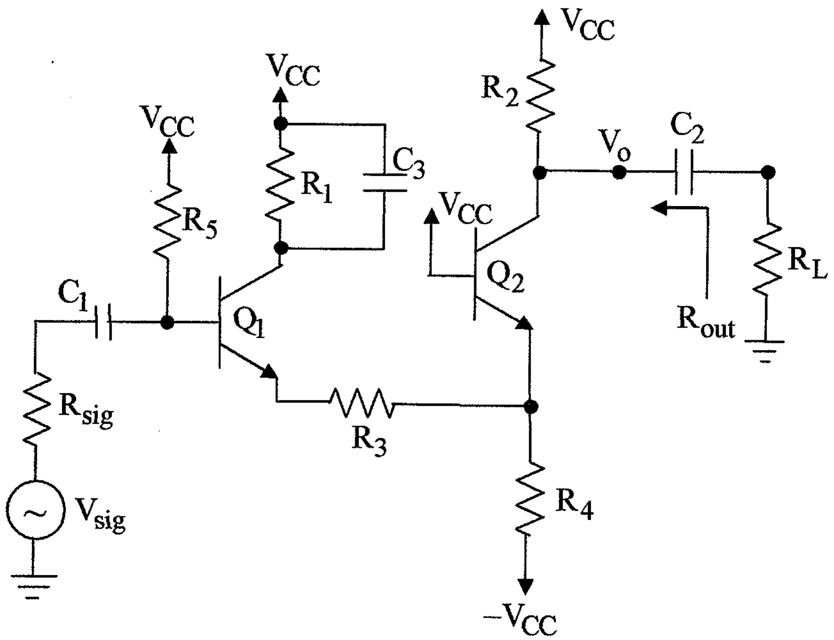


$$V_1 = \frac{V_{in} R_{eq1}}{R_{eq1} + r_{\pi 1}}$$



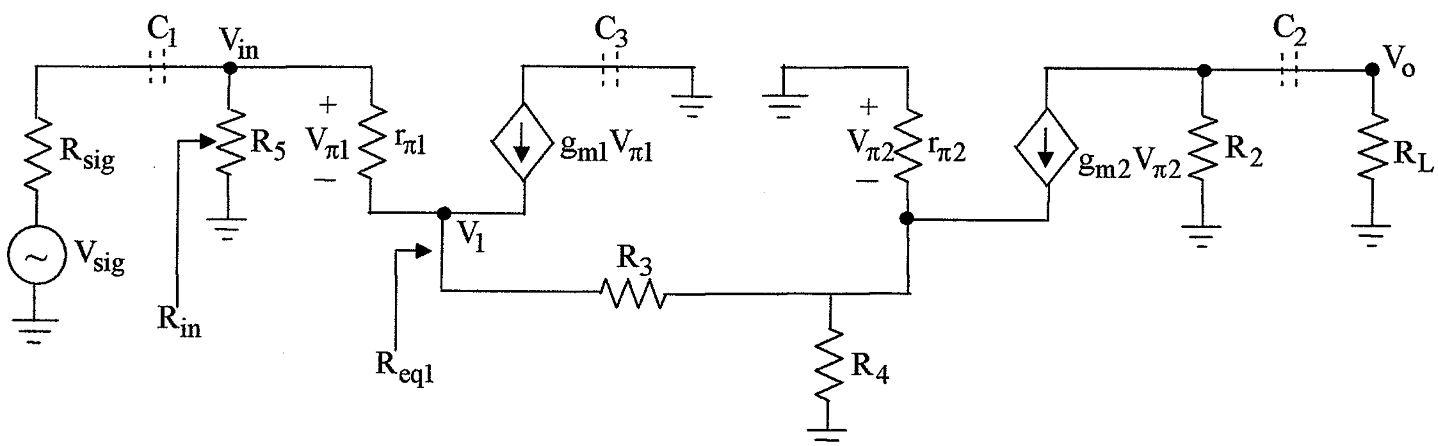
$$\left[R_3 + \left(R_4 \left\| \frac{r_{\pi 2}}{\beta+1}\right.\right)\right](\beta+1) = R_{eq1}$$

2 Stage ⇒ Common Collector/Common Base



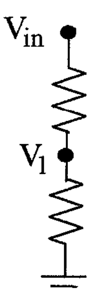
$$R_{out} = R_2$$

$$R_{in} = R_5 \left\| \left[r_{\pi 1} + \left(R_3 + R_4 \left\| \frac{r_{\pi 2}}{\beta+1} \right. \right)(\beta+1) \right] \right.$$



$$V_o = -g_{m2} V_{\pi 2} \left(R_2 \| R_L \right)$$

$$V_{\pi 2} = \frac{-V_1 \left(R_4 \left\| \frac{r_{\pi 2}}{\beta+1} \right. \right)}{\left(R_4 \left\| \frac{r_{\pi 2}}{\beta+1} \right. \right) + R_3} \qquad V_1 = \frac{V_{in} \cdot R_{eq1}}{R_{eq1} + r_{\pi 1}}$$



$$\left[R_3 + \left(R_4 \left\| \frac{r_{\pi 2}}{\beta+1} \right. \right) \right](\beta+1) = R_{eq1}$$

$$V_{in} = \frac{V_{sig} \cdot R_{in}}{R_{in} + R_{sig}}$$

$$\boxed{\frac{V_o}{V_{sig}} = \frac{g_{m2} \left(R_2 \| R_L \right) \left(R_4 \left\| \frac{r_{\pi 2}}{\beta+1} \right. \right) R_{in} \cdot R_{eq1}}{\left[\left(R_4 \left\| \frac{r_{\pi 2}}{\beta+1} \right. \right) + R_3 \right] \left[R_{in} + R_{sig} \right] \left[R_{eq1} + r_{\pi 1} \right]}}$$