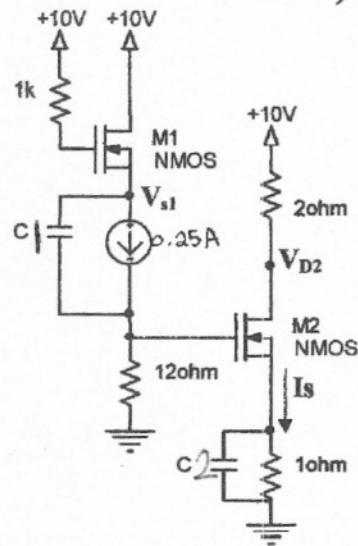


Use:  $V_t = 1V$   
 $k_n'(W/L) = 2A/V^2$   
 $\lambda = 0$  for all transistors  
 The 0.25A current source is not ideal and may have a voltage drop across it.  
 All caps are large.

Solve the circuit for the DC values of:

- (a)  $V_{D2}$
- (b)  $V_s$
- (c)  $I_s$



$V_{G1} = +10V$   
 Assume sat:  
 $I_{D1} = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (V_{GS1} - V_t)^2$   
 $0.25A = \frac{1}{2} (2) (10 - V_{S1} - 1)^2$   
 $0.25A = (9 - V_{S1})^2$   
 $\sqrt{0.25} = 9 - V_{S1}$   
 If  $V_{S1} = 8.84$   $\therefore V_{S1} = 9 \pm \sqrt{0.25} = 8.84, 9.5$   
 $V_{GS} = 10 - 8.84 = 1.16 \geq V_t$  (on)  
 If  $V_{S1} = 9.5$   $V_{GS} = 10 - 9.5 = 0.5 < V_t$  (off)

Q-pt:  $V_{GS1} = 10 - 8.84 = 1.16$

$V_{D2} = 10 - I_s(2)$   
 $V_{G2} = 10 - 1(2) = 8V$   
 $V_{DS2} = 8 - 1 = 7V \geq 1 (V_{GS} - V_t)$   
 $\therefore$  saturated

$V_{G2} = 0.25(12) = 3V$

$V_{S2} = I_s(12)$

$I_{D2} = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (3 - I_s - V_t)^2$

$I_{D2} = I_s = \frac{1}{2} (2) (2 - I_s)^2$

$I_s = (4 - 4I_s + I_s^2)$

$I_s^2 - 5I_s + 4 = 0 \Rightarrow I_s = \frac{5 \pm \sqrt{25 - 4(4)}}{2} = \frac{5 \pm 3}{2} = 4A, 1A$

if  $I_s = 4A$ :  $V_{S2} = 4V \Rightarrow V_{GS2} = 3 - 4 = -1 < V_t \therefore$  off

if  $I_s = 1A$ :  $V_{S2} = 1V \Rightarrow V_{GS2} = 3 - 1 = 2 \geq V_t \therefore$  on

Use:  $V_t = 1V$   
 $k_n'(W/L) = 1mA/V^2$   
 $v_{sig}$  is an AC source  
 Transistor 1 has DC values:  $V_{GS} = 5V, I_D = 8mA$   
 Transistor 2 has DC values:  $V_{GS} = 5V, I_D = 8mA$   
 Transistor 3 has DC values:  $V_{GS} = 3V, I_D = 2mA$   
 $\lambda = 0$  (for all transistors)

$$g_{m2} = k_n' \left(\frac{W}{L}\right) (V_{GS} - V_t) = \sqrt{2k_n' I_D} = 4m$$

$$g_{m3} = 1m(3-1) = \sqrt{2(1m)(2m)} = 2m$$

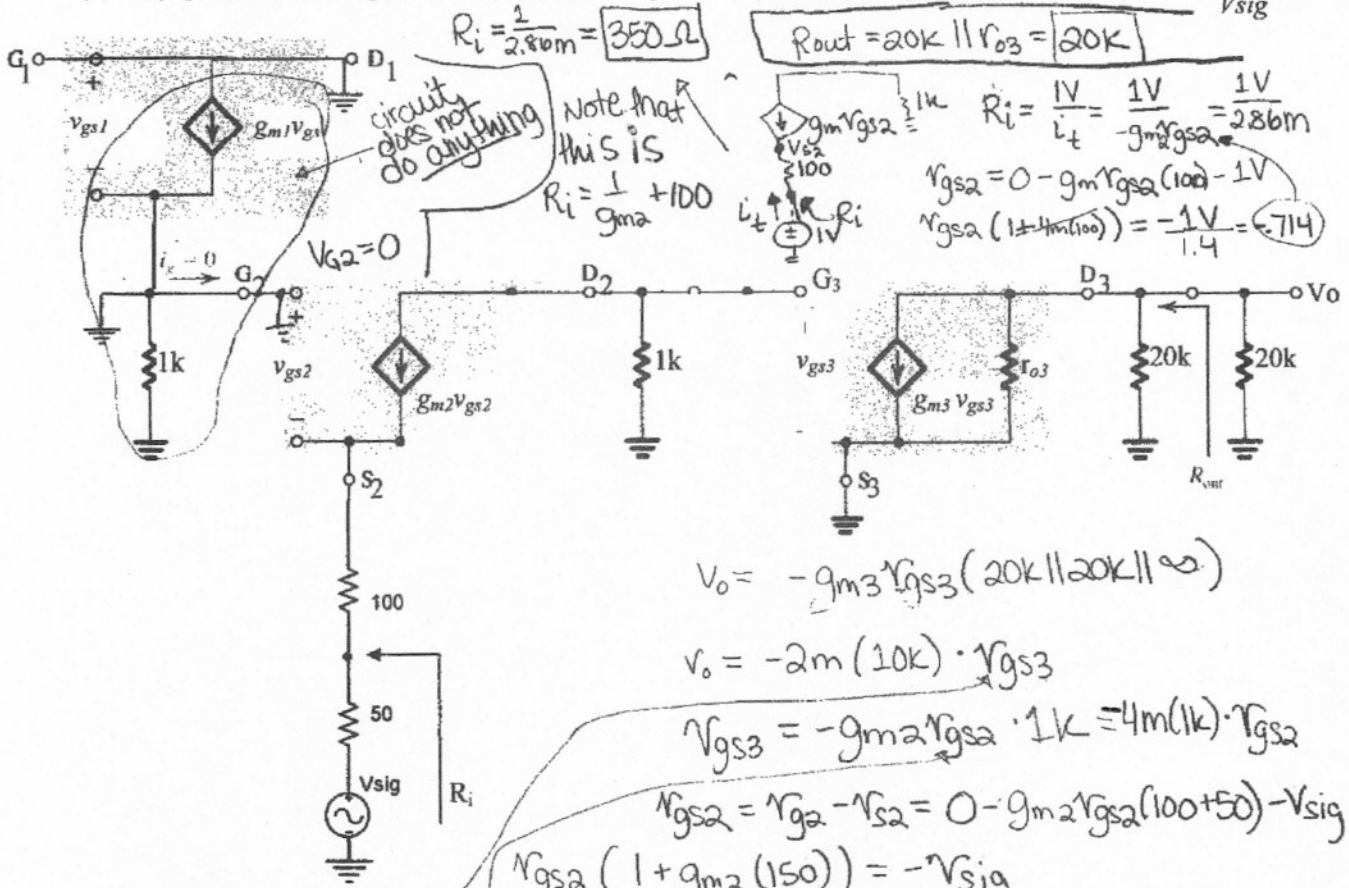
$$r_{o3} = \infty$$

For the following hybrid- $\pi$  equivalent circuit, find the following values:

(a)  $R_i$  (input resistance - ignore the 50ohm and  $V_{sig}$ )

(b)  $R_{out}$  (output resistance)

(c) gain,  $\frac{V_o}{V_{sig}}$



$$R_i = \frac{1}{2.86m} = 350 \Omega$$

$$R_{out} = 20k \parallel r_{o3} = 20k$$

$$R_i = \frac{1V}{I_t} = \frac{1V}{-g_{m2}v_{gs2}} = 286m$$

$$v_{gs2} = 0 - g_{m1}v_{gs1}(100) - 1V$$

$$v_{gs2}(1 + 4m(100)) = -\frac{1V}{1.4} = -0.714$$

$$V_o = -g_{m3}v_{gs3}(20k \parallel 20k \parallel \infty)$$

$$V_o = -2m(10k) \cdot v_{gs3}$$

$$v_{gs3} = -g_{m2}v_{gs2} \cdot 1k = 4m(1k) \cdot v_{gs2}$$

$$v_{gs2} = v_{gs2} - v_{s2} = 0 - g_{m1}v_{gs1}(100+50) - v_{sig}$$

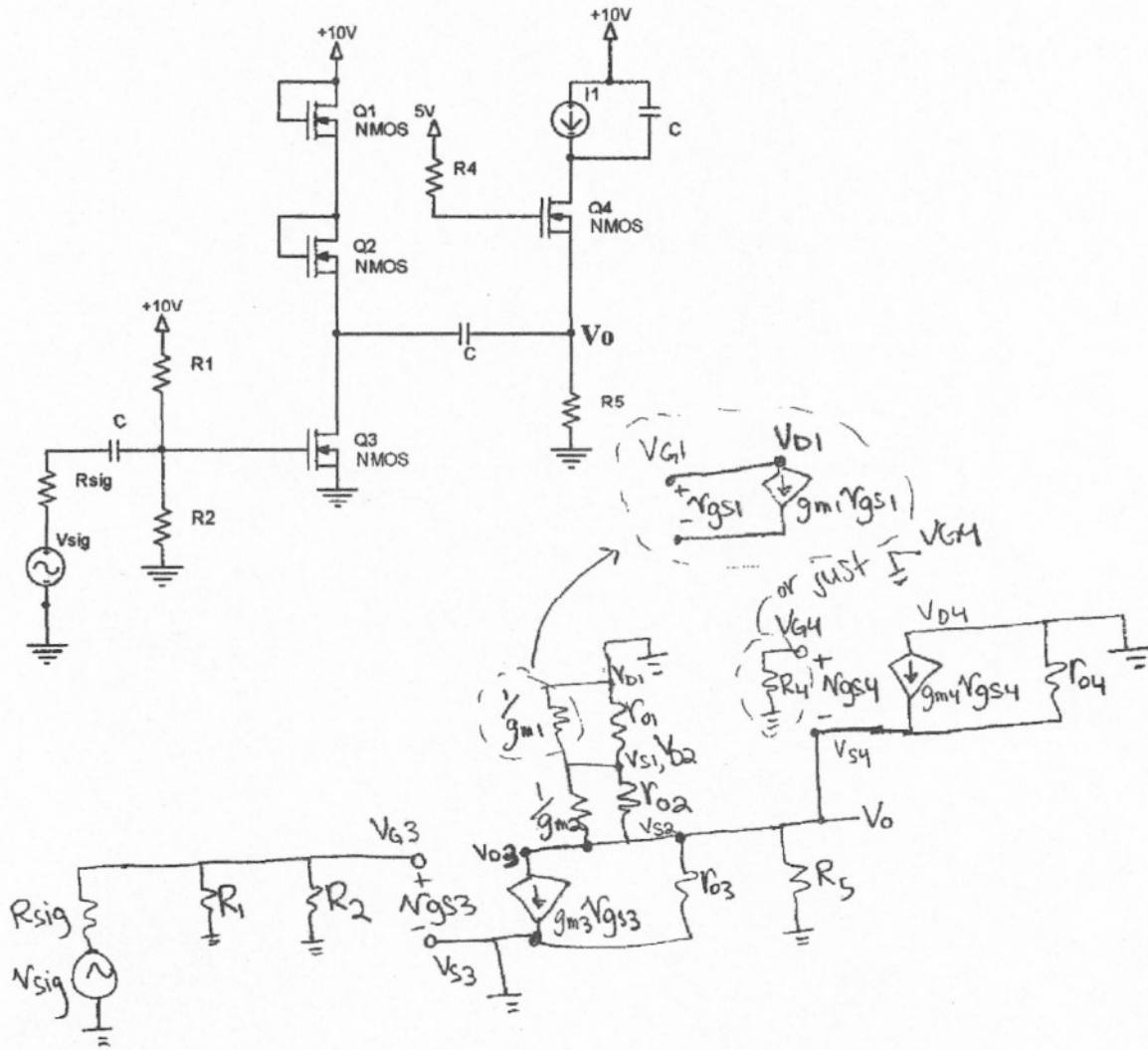
$$v_{gs2}(1 + g_{m2}(150)) = -v_{sig}$$

$$v_{gs2} = \frac{-v_{sig}}{1 + 4m(150)} = \frac{-v_{sig}}{1.6}$$

$$v_{gs3} = -4m(1k) \left( \frac{-v_{sig}}{1.6} \right) = +2.5v_{sig}$$

$$\frac{V_o}{V_{sig}} = +2.5(-2m)(10k) = -50 \frac{V}{V}$$

For the circuit shown below, draw the AC small-signal equivalent circuit (use hybrid- $\pi$  or model T). Make sure that everything is labeled in terms of the transistor number. (e.g.  $g_{m1}$ ,  $v_{gs2}$ , etc.).  $\lambda \neq 0$  for all transistors.  $v_{sig}$  is an AC source.



Let  $V_t=1V$ ,  $k_n'(W/L)=1mA/V^2$ , and  $\lambda=0$ .

(a) Solve the DC circuit assuming capacitors are acting as an open.

(b) Draw the small-signal equivalent circuit

(c) Analyze the circuit to find  $A_v=V_o/V_{in}$ ,  $R_{in}$  and  $R_{out}$

(d) Find all low frequency pole values

(e) Find  $\omega_H$  given  $C_{gs}=10pF$  and  $C_{gd}=0.1pF$ .

pole values:  $C_1 \Rightarrow \frac{1}{C_1(1k+5M)} \approx 2m \frac{rad}{sec}$

$C_2 \Rightarrow \frac{1}{C_2(6k || \frac{1}{g_m})} \approx 117 \frac{rad}{sec} \approx 19Hz$

$C_3 \Rightarrow \frac{1}{C_3(6k || 6k)} = 333 \frac{rad}{sec} = 53Hz$

$+10 - I(10Meg) + 2.5 - I(10Meg) + 2.5 = 0$

$I = \frac{15}{20Meg} = .75 \mu A$

$+V_G - I(10Meg) + 2.5 = 0$

$V_G = 7.5 - 2.5 = 5V$

$+V_S - I_S(6k) = 0$

$V_S = I_S(6k)$

$V_{GS} = 5 - I_S(6k)$

$I_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (V_{GS} - V_t)^2 = \frac{1}{2} (1m) (5 - I_D(6k) - 1)^2$

$18I_D^2 - 25I_D + 8 = 0$

$I_D = .89m, \boxed{0.5m} \rightarrow$

$V_S = 0.5m(6k) = 3V$

$V_{GS} = 5 - 3 = 2V$

$V_S = .89m(6k) = 5.34$

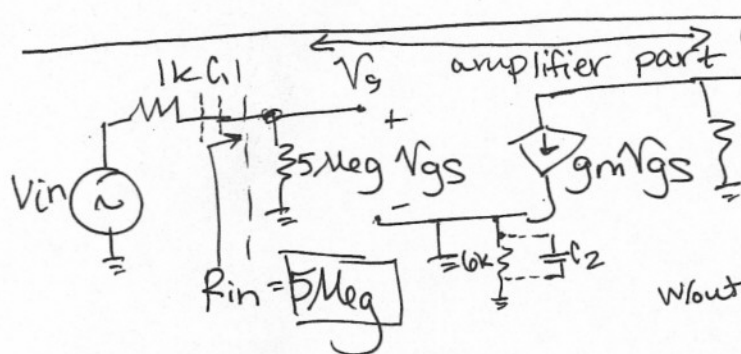
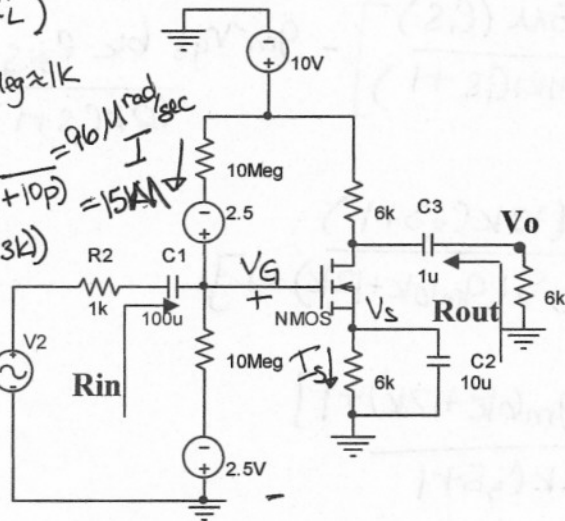
$V_D = 10 - I(6k) = 10 - 6k(.5m)$

$V_{GS} = 5 - 5.34 = -0.34 < V_t \therefore \text{OFF}$

$V_D = 7V$

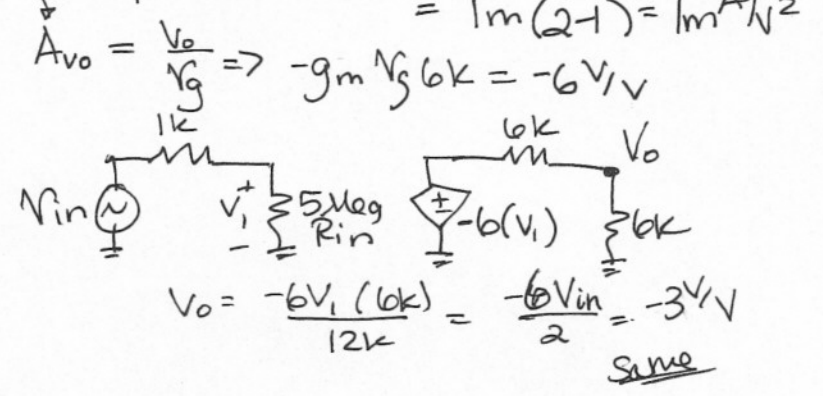
$4 = (7-3) = V_{DS} \geq (V_{GS} - V_t) = 2 - 1 = 1$

$\omega_H = \frac{1}{R_{sig}(C_{gs} + C_{gd})}$   
 $C_{gs} = C_{gd}(1 + g_m R_L)$   
 $R_{sig} = 1k || 5Meg \approx 1k$   
 $R_L = 3k$   
 $\therefore \omega_H = \frac{1}{1k(.4p + 10p)} = 15 \text{ Mrad/sec}$   
 $C_{gs} = .1p(1 + 1m(3k)) = .4p$



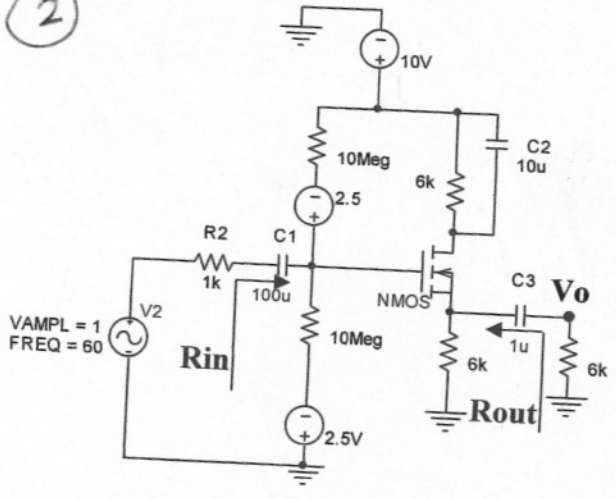
$V_o = -g_m V_{gs} (6k || 6k)$   
 $V_{gs} = V_g - 0 = \frac{V_{in}(3Meg)}{5Meg + 1k} \approx V_{in}$   
 $\therefore \frac{V_o}{V_{in}} = -g_m (3k) = -1m(3k) = \boxed{-3V/V}$

$g_m = \sqrt{2k_n'(W/L)I_D} = \sqrt{2(1m)(.5m)} = 1mA/V^2$   
 $g_m = k_n'(W/L)(V_{GS} - V_t) = 1m(2-1) = 1mA/V^2$

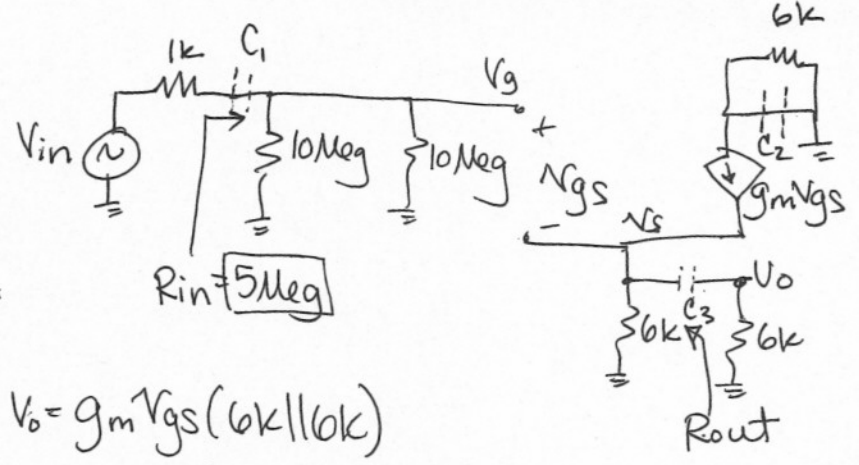




2



$I_D = .5m, V_{GS} = 2V, g_m = 1mA/V^2$



$V_{gs} = V_g - V_s$

$V_g = \frac{V_{in}(5Meg)}{5Meg + 1k} \approx V_{in}$

$V_s = g_m V_{gs} (3k)$

$V_{gs} = V_{in} - g_m V_{gs} (3k)$

$V_{gs} + g_m V_{gs} (3k) = V_{in}$

$\therefore V_{gs} = \frac{V_{in}}{(1 + g_m(3k))}$

$V_o = g_m V_{gs} (6k || 6k)$

$V_o = 1m(3k) V_{gs}$   
 $= 1m(3k) \frac{V_{in}}{4}$

$R_{out} = 6k || \frac{1}{g_m}$   
 $R_{out} = 857$

$\frac{V_o}{V_{in}} = 0.75$

$A_{vo} = \frac{V_o - V_o(6k)}{V_g} = \frac{g_m V_{gs} (6k)}{V_g}$

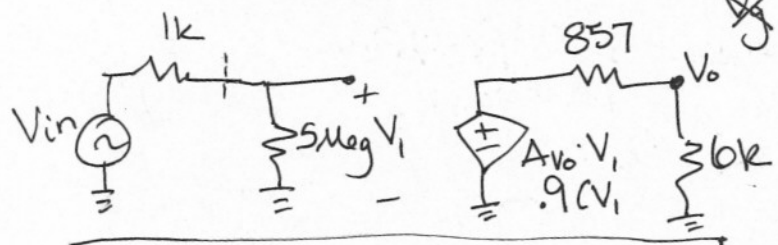
$V_{gs} = V_g - g_m V_{gs} (6k)$

$V_{gs} = \frac{V_g}{1 + (1m)(6k)} = \frac{V_g}{7}$

$\therefore \frac{V_o(-6k)}{V_g} = \frac{g_m V_g (6k)}{7} \approx 0.9 V$

$\Rightarrow V_o = A_{vo} \cdot V_i \cdot \frac{6k}{6k + 857}$

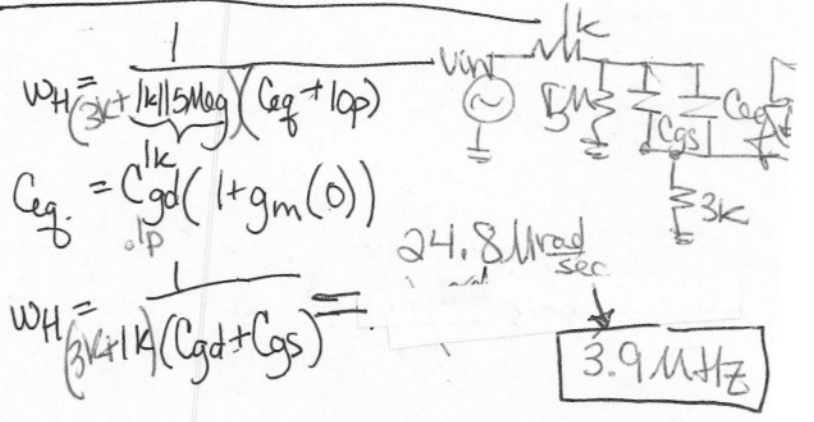
$\frac{V_o}{V_{in}} = \frac{.9 V_{in} (6k)}{6k + 857} \approx 0.79$   
same



$C_1 \Rightarrow \frac{1}{C_1(1k || 5Meg)} = 2m \text{ rad/sec}$

$C_2 \Rightarrow \frac{1}{10u(6k)} = 17 \text{ rad/sec}$

$C_3 \Rightarrow \frac{1}{1u(6k || \frac{1}{g_m} + 6k)} = 146 \text{ rad/sec}$   
 $23 \text{ Hz}$



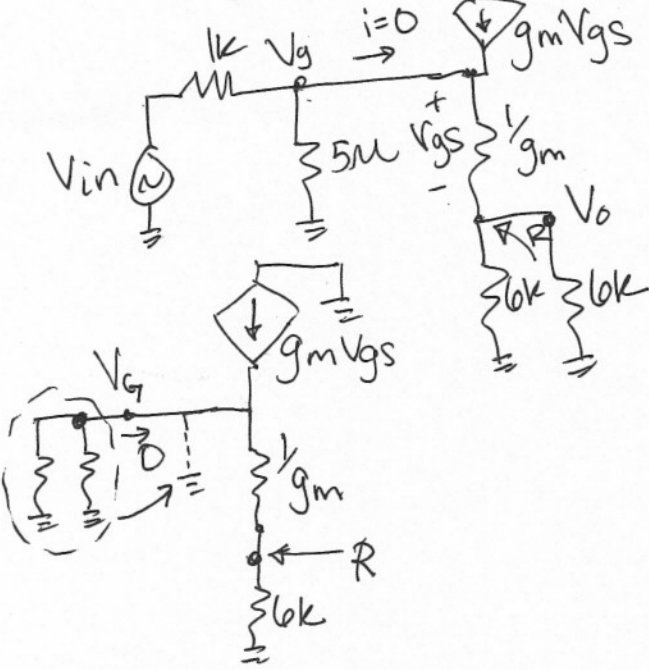
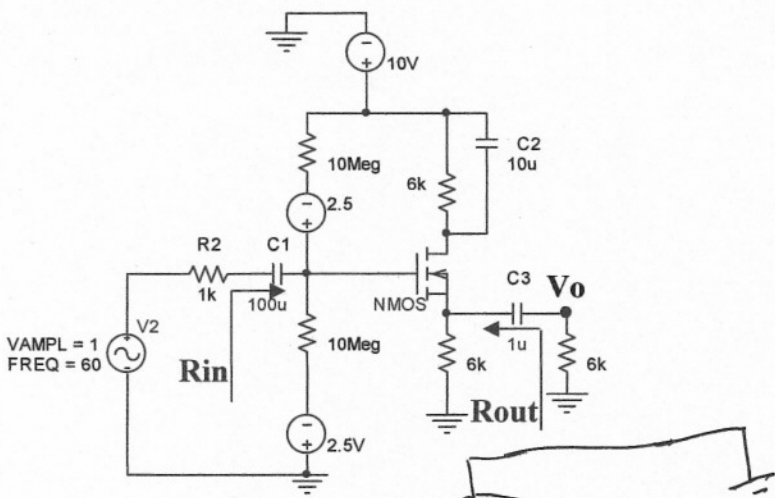
$\omega_H = \frac{1}{(3k + 1k || 5Meg)(C_{gd} + C_p)}$

$C_{gd} = C_{gd} (1 + g_m(0))$

$\omega_H = \frac{1}{(3k + 1k)(C_{gd} + C_{gs})}$

$24.8 \mu\text{rad/sec}$

$3.9 \text{ MHz}$



$$V_g \approx V_{in}$$

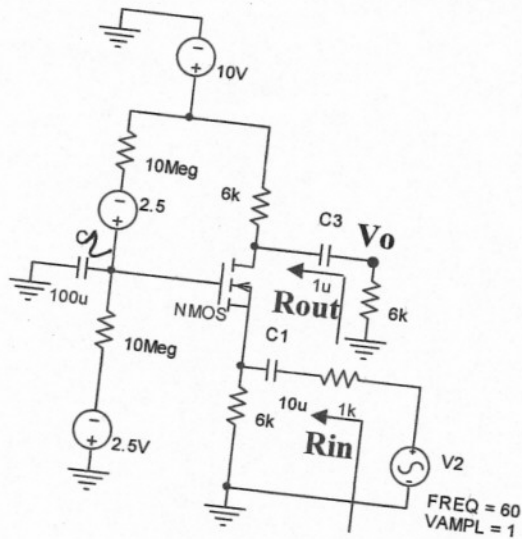
$$V_o = g_m V_{gs} (3k)$$

$$V_{gs} = V_g - V_s = V_{in} - g_m V_{gs} (3k)$$

$$V_{gs} + g_m V_{gs} (3k) = V_{in}$$

$$V_{gs} = \frac{V_{in}}{(1 + g_m (3k))} = \frac{V_{in}}{4}$$

$$\frac{V_o}{V_{in}} = \frac{3}{4} \frac{V}{V}$$



$$g_m = 1 \text{ mA/V}^2$$

$$V_o = -g_m V_{gs} (3k)$$

$$V_{gs} = V_g - V_s = 0 - V_s$$

$$g_m(-V_s) + \frac{V_{in} - V_s}{1k} + \frac{-V_s}{6k} = 0$$

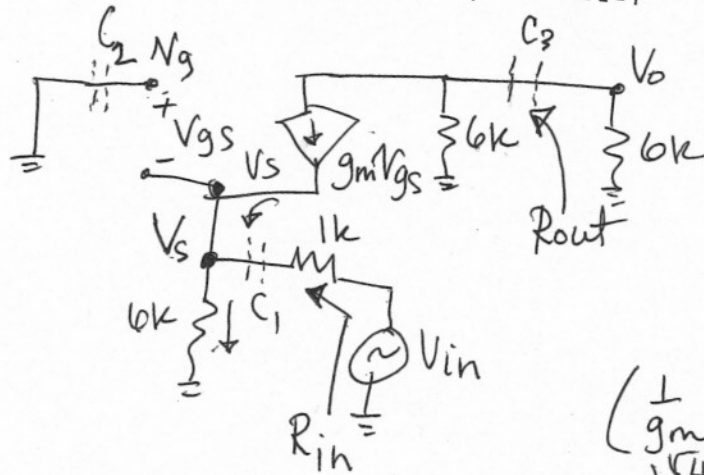
$$+V_s \left( g_m + \frac{1}{1k} + \frac{1}{6k} \right) = \frac{V_{in}}{1k}$$

$$V_s = \frac{V_{in} \left( \frac{1}{1k} \right)}{g_m + \frac{1}{1k} + \frac{1}{6k}}$$

$$\frac{V_s}{V_{in}} = \frac{1}{1k(2.2 \text{m})} = 0.45$$

$$\frac{V_s}{V_{in}} = \boxed{.46 \text{ V/V}}$$

$$\frac{V_o}{V_{in}} = 3(.45) = \boxed{1.35 \text{ V/V}}$$



$$\left( \frac{1}{g_m} \parallel 6k \parallel 1k \right) \approx 461$$

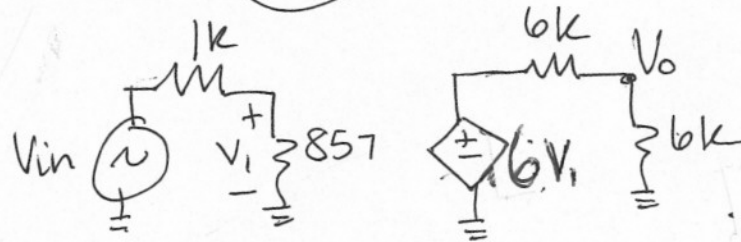
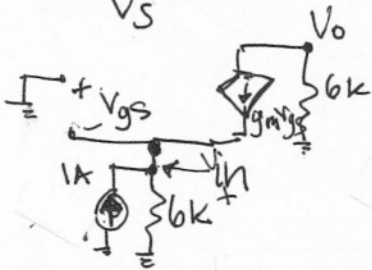
$$R_{out} = 6k \quad R_{in} = 6k \parallel \frac{1}{g_m} = \boxed{857}$$

$$A_{vo} \Rightarrow \frac{V_o}{V_s}$$

$$V_o = -g_m V_{gs} (6k)$$

$$V_{gs} = V_g - V_s = -V_s$$

$$\frac{V_o}{V_s} = +g_m (6k) = 6$$



$$V_1 = \frac{V_{in} (857)}{1,857} = V_{in} (.46)$$

$$V_o = \left( \frac{6k}{12k} \right) V_1 = 3V_1 = \boxed{3(.46) V_{in}}$$

$$\frac{V_o}{V_{in}} = \boxed{1.38 \text{ V/V}}$$

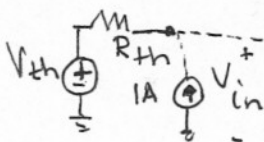
$$R_{th} = 6k \parallel \frac{1}{g_m} = 857$$

$$V_{in} = R_{th} I_A + V_{th}$$

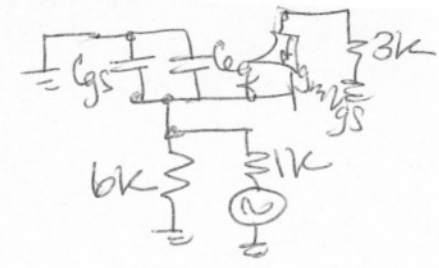
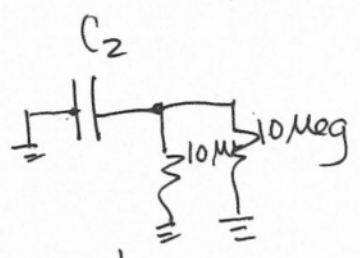
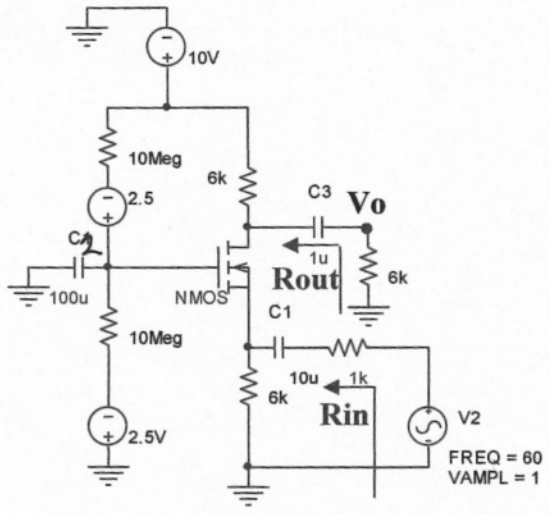
$$V_{th} = (V_{in} - R_{th})$$

$$I_A - \frac{V_{in}}{6k} + g_m(-V_{in}) = 0$$

$$V_{in} = \frac{1}{\frac{1}{6k} + g_m} = 857$$



$$\frac{V_{in} - V_{th}}{R_{th}} = 1$$



$$C_2 (5 \text{Meg}) = 2 \text{m} \frac{\text{rad}}{\text{sec}}$$

$$C_3 (6\text{k} + 6\text{k}) = 83 \frac{\text{rad}}{\text{sec}} \approx 13 \text{Hz}$$

$$C_1 (1\text{k} + 6\text{k} \parallel \frac{1}{g_m}) = 18.57 \text{m}$$

$$R_L = 3\text{k}$$

$$R_{sig} = 857$$

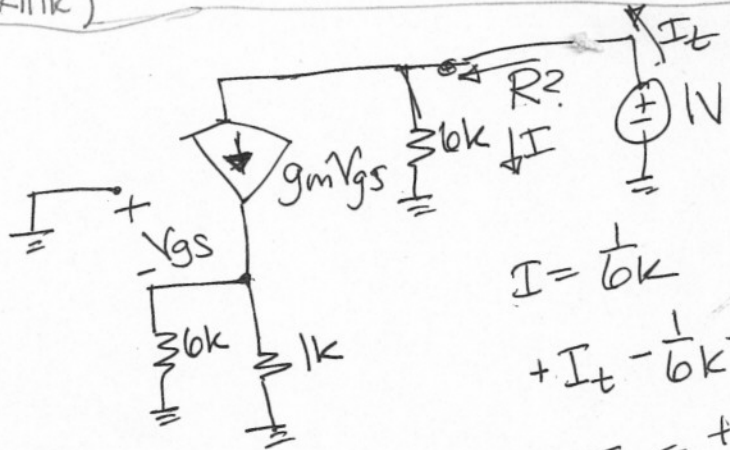
$$\omega_H = 112 \text{M} \frac{\text{rad}}{\text{sec}}$$

$$\boxed{17.9 \text{MHz}}$$

$$\omega_H = \frac{1}{R_{sig}' (C_{eq} + C_{gs})}$$

$$(0 + 6\text{k} \parallel 1\text{k})$$

$$C_{eq} = C_{gd}(1 + g_m R_L) = 4\text{p}$$



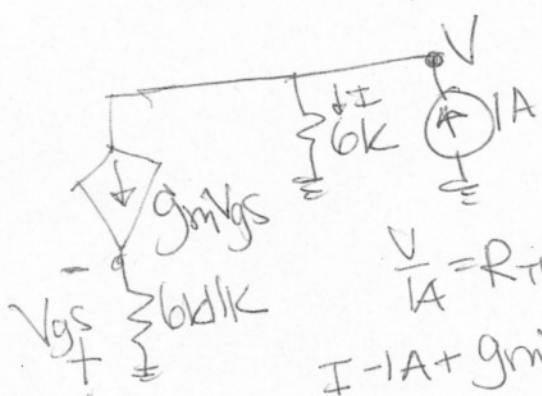
$$\frac{V}{I_t} = R_{th}$$

$$R_{th} = \frac{1\text{V}}{\frac{1}{6\text{k}}} = 6\text{k}$$

$$I = \frac{1}{6\text{k}}$$

$$+I_t - \frac{1}{6\text{k}} - g_m V_{gs} = 0$$

$$I_t = +\frac{1}{6\text{k}} + g_m V_{gs}$$



$$\frac{V}{1\text{A}} = R_{th}$$

$$I - 1\text{A} + g_m V_{gs} = 0$$

$$0 = -V_{gs} = g_m V_{gs} (6\text{k} \parallel 1\text{k}) + V_{gs}$$

$$\therefore V_{gs} = 0$$



Easiest way to see

$$i_{sc} = -g_m V_{gs}$$

$$V_{th} = -g_m V_{gs} (6\text{k})$$

$$R_{th} = \frac{V_{th}}{i_{sc}} = \frac{-g_m V_{gs} (6\text{k})}{-g_m V_{gs}} = 6\text{k}$$