7-6.3 Decibel Scale for Power Ratios

The unit for power P is watts (W). In many engineering problems, the quantity of interest is the ratio of two power levels, P_1 and P_2, such as the incident and reflected powers on a transmission line, and often the ratio P_1/P_2 may vary over several orders of magnitude. The decibel (dB) scale is logarithmic, thereby providing a convenient representation of the power ratio, particularly when numerical values of P_1/P_2 are plotted against some variable of interest. If

$$G = \frac{P_1}{P_2}, \qquad (7.110)$$

then

$$G \text{ [dB]} \triangleq 10 \log G = 10 \log \left(\frac{P_1}{P_2} \right) \quad \text{(dB)}. \qquad (7.111)$$

Table 7-2 provides a comparison between some values of G and the corresponding values of G [dB]. Even though decibels are defined for power ratios, they can sometimes be used to represent other quantities. For example, if $P_1 = V_1^2/R$ is the power dissipated in a resistor R with voltage V_1 across it at time t_1, and $P_2 = V_2^2/R$ is the power dissipated in the same resistor at time t_2, then

$$\begin{aligned} G \text{ [dB]} &= 10 \log \left(\frac{P_1}{P_2} \right) \\ &= 10 \log \left(\frac{V_1^2/R}{V_2^2/R} \right) \\ &= 20 \log \left(\frac{V_1}{V_2} \right) \\ &= 20 \log(g) \triangleq g \text{ [dB]}, \end{aligned} \qquad (7.112)$$

where $g = V_1/V_2$ is the voltage ratio. *Note that for voltage (or current) ratios the scale factor is 20 rather than 10,* which results in G [dB] $= g$ [dB].

Table 7-2: Power ratios in natural numbers and in decibels.

G	G [dB]
10^x	$10x$ dB
4	6 dB
2	3 dB
1	0 dB
0.5	−3 dB
0.25	−6 dB
0.1	−10 dB
10^{-3}	−30 dB

The ***attenuation rate***, representing the rate of decrease of the magnitude of $\mathbf{S}_{\text{av}}(z)$ as a function of propagation distance, is defined as

$$\begin{aligned} A &= 10 \log \left[\frac{S_{\text{av}}(z)}{S_{\text{av}}(0)} \right] \\ &= 10 \log(e^{-2\alpha z}) \\ &= -20\alpha z \log e \\ &= -8.68\alpha z = -\alpha \text{ [dB/m] } z \quad \text{(dB)}, \end{aligned} \qquad (7.113)$$

where

$$\alpha \text{ [dB/m]} \triangleq 8.68\alpha \text{ [Np/m]}. \qquad (7.114)$$

We also note that, since $\mathbf{S}_{\text{av}}(z)$ is directly proportional to $|\mathbf{E}(z)|^2$,

$$A = 10 \log \left[\frac{|E(z)|^2}{|E(0)|^2} \right] = 20 \log \left[\frac{|E(z)|}{|E(0)|} \right] \quad \text{(dB)}. \qquad (7.115)$$

Example 7-6 Power Received by a Submarine Antenna

A submarine at a depth of 200 m uses a wire antenna to receive signal transmissions at 1 kHz. Determine the power density incident upon the submarine antenna due to the EM wave of Example 7-4.

Solution: From Example 7-4, $|E_0| = |E_{x0}| = 4.44$ (mV/m), $\alpha = 0.126$ (Np/m), and $\eta_c = 0.044\angle 45°$ (Ω). Application of Eq. (7.109) gives

$$\begin{aligned} \mathbf{S}_{av}(z) &= \hat{\mathbf{z}} \frac{|E_0|^2}{2|\eta_c|} e^{-2\alpha z} \cos\theta_\eta \\ &= \hat{\mathbf{z}} \frac{(4.44 \times 10^{-3})^2}{2 \times 0.044} e^{-0.252z} \cos 45° \\ &= \hat{\mathbf{z}} 0.16 e^{-0.252z} \qquad (\text{mW/m}^2). \end{aligned}$$

At $z = 200$ m, the incident power density is

$$\begin{aligned} \mathbf{S}_{av} &= \hat{\mathbf{z}}(0.16 \times 10^{-3} e^{-0.252 \times 200}) \\ &= 2.1 \times 10^{-26} \qquad (\text{W/m}^2). \end{aligned}$$ ■

EXERCISE 7.10 Convert the following values of the power ratio G from natural numbers to decibels: (a) 2.3, (b) 4×10^3, (c) 3×10^{-2}.

Ans. (a) 3.6 dB, (b) 36 dB, (c) −15.2 dB. (See ✱)

EXERCISE 7.11 Find the voltage ratio g in natural units corresponding to the following decibel values of the power ratio G: (a) 23 dB, (b) −14 dB, (c) −3.6 dB.

Ans. (a) 14.13, (b) 0.2, (c) 0.66. (See ✱)

CHAPTER HIGHLIGHTS

- A spherical wave radiated by a source becomes approximately a uniform plane wave at large distances from the source.
- The electric and magnetic fields of a transverse electromagnetic (TEM) wave are orthogonal to each other, and both are perpendicular to the direction of wave travel.
- The magnitudes of the electric and magnetic fields of a TEM wave are related by the intrinsic impedance of the medium.
- Wave polarization describes the shape and locus of the tip of the **E** vector at a given point in space as a function of time. The polarization state, which may be linear, circular, or elliptical, is governed by the ratio of the magnitudes of and the difference in phase between the two orthogonal components of the electric field vector.
- Media are classified as lossless, low-loss, quasi-conducting, or good conducting on the basis of the ratio $\varepsilon''/\varepsilon' = \sigma/\omega\varepsilon$.
- Unlike the d-c case, wherein the current flowing through a wire is distributed uniformly across its cross section, in the a-c case most of the current is concentrated along the outer perimeter of the wire.
- Power density carried by a plane EM wave traveling in an unbounded medium is akin to the power carried by the voltage/current wave on a transmission line.

GLOSSARY OF IMPORTANT TERMS

Provide definitions or explain the meaning of the following terms:

guided medium
unbounded media
spherical wave
uniform plane wave
complex permittivity ε_c
wavenumber k
TEM wave
intrinsic impedance η
wave polarization
elliptical polarization
circular polarization
linear polarization
LHC and RHC polarizations
attenuation constant α
phase constant β
skin depth δ_s
low-loss dielectric
quasi-conductor
good conductor