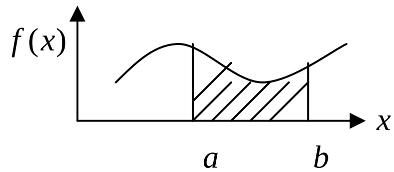
a) TRAPEZOIDAL INTEGRATION

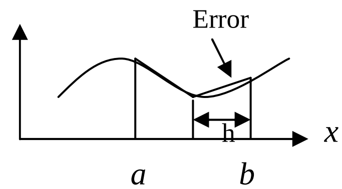


$$\int_a^b f(x) \quad dx$$

= Area under curve

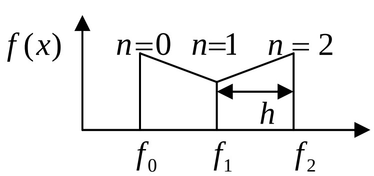
Numerical Integration approximates the curve by a function and integrates this new approximate function.

Trapezoidal integration approximates the function by a line (1st order polynomials).



Straight lines are approximations of curved function f(x)

Calculate the area under each trapezoid



$$\text{Area}_1 = \left(\frac{f_0 + f_1}{2}\right)h$$

$$\text{Area}_2 = \left(\frac{f_1 + f_2}{2}\right)h$$

$$\text{Area} = \left(\frac{f_0}{2} + f_1 + \frac{f_2}{2}\right)h$$

TRAPEZOIDAL RULE

$$\int_a^b f(x) \quad dx = h\left[\frac{f(a)}{2} + \frac{f(b)}{2} + \sum_{j=1}^{n-1} f(a+jh)\right] + Error \quad \bullet\left(h_3 f''\right)$$

Assuming equal-spaced points

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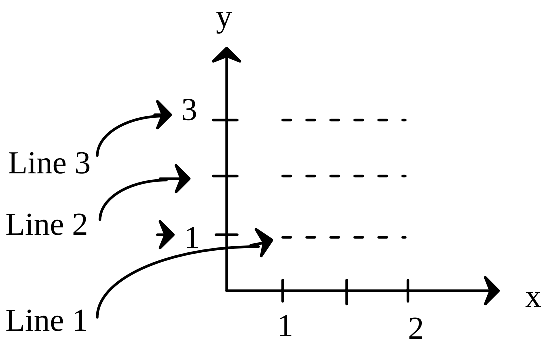
2-D INTEGRATION (TRAPEZOIDAL METHOD, useful for plates, not used in this lab, but included here for your interest and amusement)



$$f(x,y) = xy$$

$$\int_1^2 \int_1^3 (xy)dxdy =$$

Analytical
6.0

Let

$n_x = n_y = 2$ $\qquad$ $h_x = (2 - 1)/2 = 0.5$

Line 1) $y = 1$ $\qquad$ $f_1(x) = x$ $\qquad$ $h_y = (3 - 1)/2 = 1$

$$\int_1^2 f_1(x)\,dx = \int_1^2 x\,dx = \left[f_1(1) + 2f_1\left(\frac{3}{2}\right) + f_1(2) \right] \frac{h_x}{2}$$

$$= \left[1 + 2\left(\frac{3}{2}\right) + 2 \right] \frac{h_x}{2} = 6\frac{h_x}{2} = \frac{3}{2} = g(y=1) = g(1)$$

Line 2) $y = 2$ $\qquad$ $f_2(x) = 2x$

$$\int_1^2 f_2(x)\,dx = \int_1^2 2x\,dx = \left[f_2(1) + 2\,f_2\left(\frac{3}{2}\right) + f_2(2) \right] \frac{h_x}{2}$$

$$= \left[2 + 4\left(\frac{3}{2}\right) + 4 \right] \frac{h_x}{2} = 6\,h_x = 3 = g(2)$$

Line 3) $y = 3$ $\qquad$ $f_3(x) = 3x$

$$\int_1^2 f_3(x)\,dx = \int_1^2 3x\,dx = \left[f_3(1) + 2f_3\left(\frac{3}{2}\right) + f_3(2) \right]$$

$$= \left[3 + 2\left(\frac{9}{2}\right) + 6 \right] \frac{h_x}{2} = \frac{9}{2} = g(3)$$

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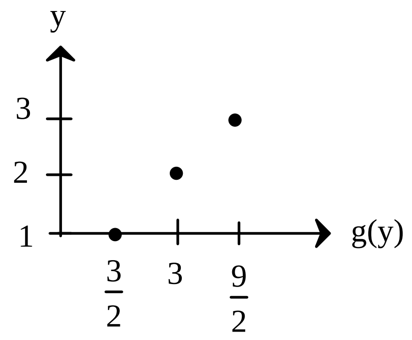
$$\text{ERROR}_x \cong h_x \frac{\partial^2 f}{\partial x^2} = 0$$



g(y):

g(1) = 3/2

g(2) = 3

G(3) = 9/2

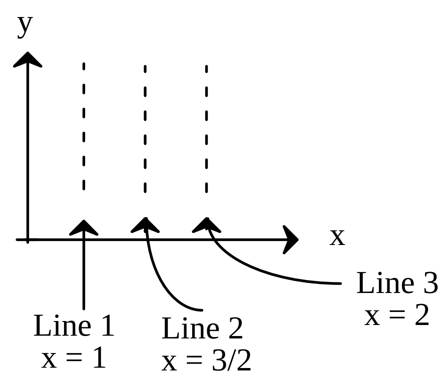
$$\int_1^3 g(y)\,dy = [g(1) + 2g(2) + g(3)]\frac{h_y}{2}$$

$$= \left[\frac{3}{2} + 2(3) + \frac{9}{2}\right]\frac{1}{2} = \boxed{6 \text{ Numerical}}$$

$$\text{ERROR}_y = h_y \frac{\partial^2 f}{\partial y^2} = 0$$

$$\text{ERROR} = \text{ERROR}_x + \text{ERROR}_y = 0$$

Note: You would have obtained identical results by:



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