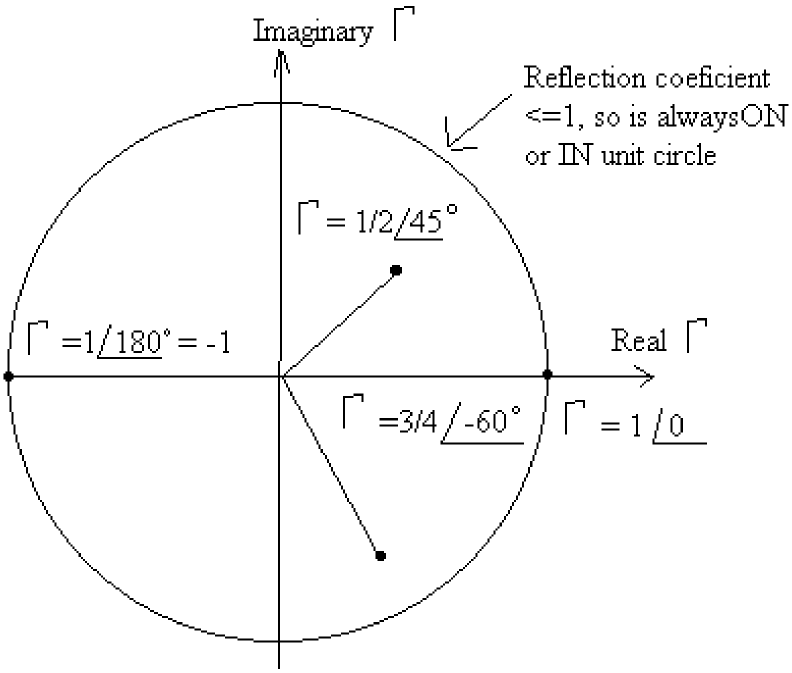
ECE 3300 SMITH CHARTS

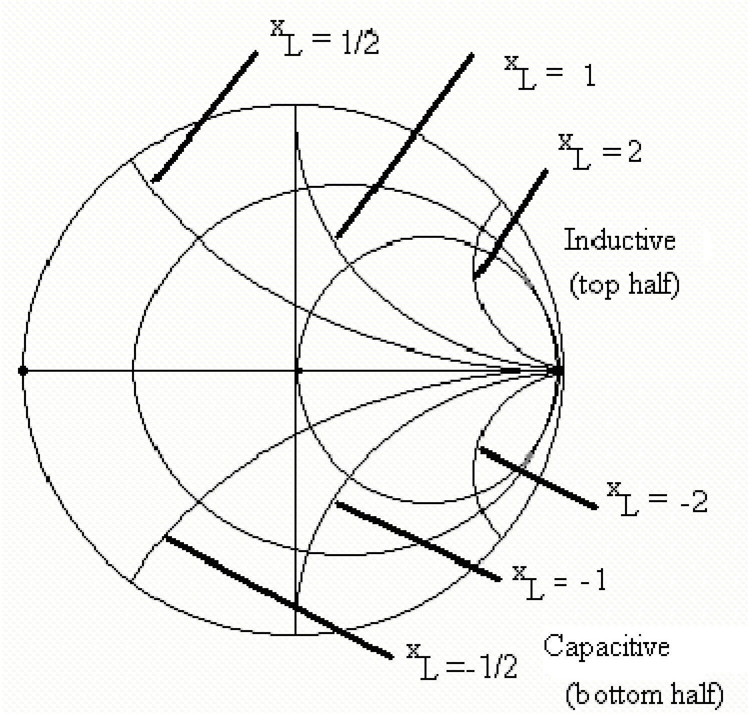
Smith Chart Circles:
A Smith chart is a graphical representation of the complex reflection coefficient, Γ

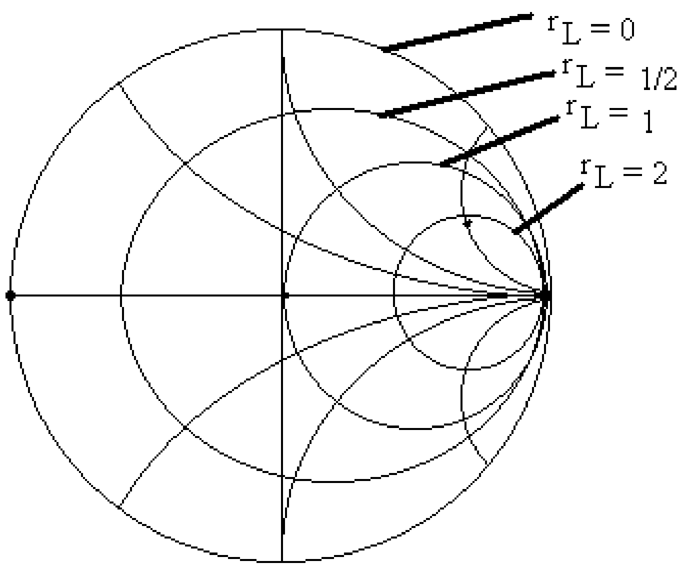


Smith Chart for Reflection Coefficient and Load Impedance:
Reflection Coefficient and Load (Z_L) are directly related:

$\Gamma = (Z_L / Zo - 1) / (Z_L/Zo+ 1) = (z_L - 1) / (z_L + 1)$
OR
$Z_L / Zo = z_L = (1 + \Gamma) / (1 - \Gamma)$ ← This is NORMALIZED load impedance
$z_L = r_L + j\, x_L$

The real and imaginary parts of z_L are functions of Γ, and these functions can be plotted on the same chart. Remember $|\Gamma| \leq 1$.





Example: Given Z_L , find Γ using Smith Chart
See transparencies (Copies to be made available in copy room)
How to find Γ :

1) Find Normalized load Impedance, $z_L = Z_L / Zo = r_L + j\ x_L$
2) Find intercept of semicircles for r_L and x_L and PLOT z_L
3) Draw line from center of smith chart to (or through) z_L
4) Read angle of Γ from outside of Smith chart
5) Measure $|\Gamma|$ with a protractor and compare to line on bottom of smith chart labeled “Ref. Coeff. E or I”

$Zo = 100$ ohms

Z_L = open circuit

1) $z_L = \infty = \infty + j\,0$
2) PLOT (far right)
3) Draw Line through z_L. Read $\angle 0$
4) Measure using a protractor (or this one is obviously =1) $|\Gamma| = 1$

$\Gamma = 1 \angle 0$ (which is what we expect for an open circuit)

Z_L = short circuit

1) $z_L = 0 = 0 + j\,0$
2) PLOT (far left)
3) Draw Line through z_L. Read $\angle 180°$
4) Measure using a protractor (or this one is obviously =1) $|\Gamma| = 1$

$\Gamma = 1 \angle 180° = -1$ (which is what we expect for an short circuit)

$Z_L = 100 + j\,0$ ohms

1) $z_L = Z_L / Zo = 1 + j\,0$
2) PLOT (center of smith chart)
3) Draw Line through z_L. Not so easy … $\angle$?
4) Measure using a protractor (or this one is obviously =0) $|\Gamma| = 0$

$\Gamma = 0 \angle$? (which is what we expect for a matched load)

$Z_L = 100 + j\,100$ ohms

1) $z_L = Z_L / Zo = 1 + j\,1$
2) PLOT (top right quadrant)
3) Draw Line through z_L. about $\angle 63°$
4) Measure using a protractor $|\Gamma| = 0.45$

$\Gamma = 0.45 \angle 63°$

$\Gamma = (z_L - 1) / (z_L + 1) = (0+j1) / (2+j1) = 1\angle 90° / 2.236 \angle 26.56° = 0.45 \angle 63.43°$

How do you find load impedance if given Γ?

1) Plot Γ
2) Read $z_L = z_L + j\, z_L$
3) Unnormalize: $Z_L = z_L * Zo$

Admittance vs. Impedance:

Admittance $y_L = 1 / z_L$

$\Gamma = (z_L - 1) / (z_L + 1) = (1/y_L - 1) / (1/y_L + 1) = - (y_L - 1) / (y_L + 1)$ = 180 out of phase

Steps to find Γ from y_L:

1) Find normalized $y_L = Zo / Z_L = g_L + jb_L$
2) Plot it (Using same curves g=r and b=x)
3) "Transform it through the origin" … Rotate 180 degrees = draw a line of equal length through the origin. Now you have found z_L
4) Read Γ as before

EXAMPLE (See transparencies)

Input Impedance:

$Z_{in} = Zo\,[1 + \Gamma e^{-j2\beta l}] / [1 - \Gamma e^{-j2\beta l}]$

$z_{in} = Z_{in} / Zo = [1 + \Gamma e^{-j2\beta l}] / [1 - \Gamma e^{-j2\beta l}]$

Define reflection coefficient at the input (NOT Γg) as the reflection coefficient looking into the load frm the input location. $\Gamma_l = \Gamma_L \angle -2\beta l$

This represents moving $2\beta l$ radians towards the generator.

You can convert this distance to degrees, and read it off the outer circles on the Smith Chart (notice DIRECTION to the generator is marked)

OR $2\beta l = 2(2\pi / \lambda)\, l = 4\pi\, (l / \lambda)$ This has been normalized for you on the outside circle around the Smith Chart. Observe that if $l=\lambda$, this represents 2 complete rotations around the Smith Chart. $L = \lambda/2$ represents one complete rotation.

Does this make sense? For a Transmission line of length $L = \lambda/2$, traveling from generator to the load and back would represent a phase shift of 360 degrees … one complete rotation.

Then $z_{in} = [1 + \Gamma_l] / [1 - \Gamma_l]$

How to find Zin :

1) Normalize $z_L = Z_L / Zo$
2) Plot z_L. This also gives you Γ_L.
3) Rotate Γ distance l (given in wavelengths) TOWARDS the generator.
4) Read z_{in} and Γ_l
5) $Zin = z_{in} * Zo$

EXAMPLE (see transparencies)

Standing Wave Ratio:

To read SWR from the Smith Chart:

1) PLOT z_L

2) Draw a circle through it.
3) Read SWR from real axis to right (SWR ≥ 1)

EXAMPLE (See transparencies)

Voltage Minima and Maxima:
To read Voltage maxima off Smith Chart:
1) PLOT z_L
2) First Voltage maximum occurs on right side of real axis. First Voltage minimum occurs on left side of real axis.

EXAMPLE (See transparencies)