ECE 3300 Gradient of a Scalar Field

GRADIENT:

You have seen the gradient in calculus:

$$\nabla T = \bar{x}\frac{\partial T}{\partial x} + \bar{y}\frac{\partial T}{\partial y} + \bar{z}\frac{\partial T}{\partial z}$$

This is a VECTOR ∇ operating on a scalar variable T.

The Gradient OPERATOR is a vector:

$$\nabla = \bar{x}\frac{\partial}{\partial x} + \bar{y}\frac{\partial}{\partial y} + \bar{z}\frac{\partial}{\partial z}$$

Physical Meaning:
It gives the change of T (derivative of T) in the x,y,z directions.
By combining them into a vector, additional information is obtained for a scalar which varies in 3D.
magnitude = maximum rate of change in any direction
direction = direction of maximum increase

Directional Derivative:
Given a scalar function which varies in 3D, find out how much it varies in some specific direction.
Directional Derivative = Change in T / Change in location (direction)

$$= \frac{dT}{dl} = \nabla T \bullet \overline{a_l}$$

Example: Find out how much the temperature varies across the inlet to a jet engine. It is observed that T= sin(x)sin(y), and the inlet is a slot in the x-direction, at y=π/2.

$$\nabla T = \bar{x}\frac{\partial T}{\partial x} + \bar{y}\frac{\partial T}{\partial y} + \bar{z}\frac{\partial T}{\partial z} = \bar{x}\cos(x) + \bar{y}\cos(y) + \bar{z}0$$

$\mathbf{a}_l = \mathbf{x}$

$$\frac{dT}{dl} = \nabla T \bullet \overline{a_l} = (\bar{x}\cos(x) + \bar{y}\cos(y)) \bullet \bar{x} = \cos(x)$$

The change in temperature across the inlet depend on where (x) on the inlet you are!

Gradient in Cylindrical Coordinates:

Conversion from rectangular to cylindrical requires:
1) Change all variables (∂T/∂x, ∂T/∂y, ∂T/∂z)
2) Change all vectors (**x,y,z**)

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial T}{\partial \phi}\frac{\partial \phi}{\partial x} + \frac{\partial T}{\partial z r}\frac{\partial z}{\partial x}$$

Your book does this for dT/dx. Here is dT/dy:

$$r = \sqrt{x^2 + y^2}$$

$$\frac{\partial r}{\partial y} = (2y)(1/2)\left(\sqrt{x^2 + y^2}\right)^{-1} = \frac{y}{\sqrt{x^2 + y^2}} = \frac{y}{r} = \sin(\phi)$$

$$\phi = \arctan(y/x)$$

$$\frac{\partial \phi}{\partial y} =$$

$\partial z / \partial y = 0$

So: $\frac{\partial T}{\partial y} = \frac{\partial T}{\partial r}\sin\phi - \frac{\partial T}{\partial \phi}$?

Do similar equations for dT/dx (in your book) and dT/dz,

2) Use relations for **x,y,z** in Table 3-2
3) Substitute into ∇ equation:

$$\nabla = \bar{r}\frac{\partial}{\partial r} + \bar{\phi}\frac{1}{r}\frac{\partial}{\phi} + \bar{z}\frac{\partial}{\partial z}$$

Gradient in Spherical Coordinates:

$$\nabla = \bar{R}\frac{\partial}{\partial R} + \bar{\theta}\frac{1}{R}\frac{1}{\partial \theta} + \bar{\phi}\frac{1}{R\sin\theta}\frac{\partial}{\partial \phi}$$

Properties of the Gradient:

1) $\nabla(U+V)=\nabla U+\nabla V$ ← Linearity
2) $\nabla(UV)=U\nabla V+V\nabla U$ ← Used Frequently in Derivations of Numerical Simulations
3) $\nabla V^n = n V^{n-1} \nabla V$

Gradient is used to find the electric field if you have the voltage:

$$\bar{E} = -\nabla V$$

How do you find the voltage from the electric field? (THIS applies to the Biot Savart lab)

$$V_{AB} = V_A - V_B = \int_A^B \bar{E} \bullet d\ell$$