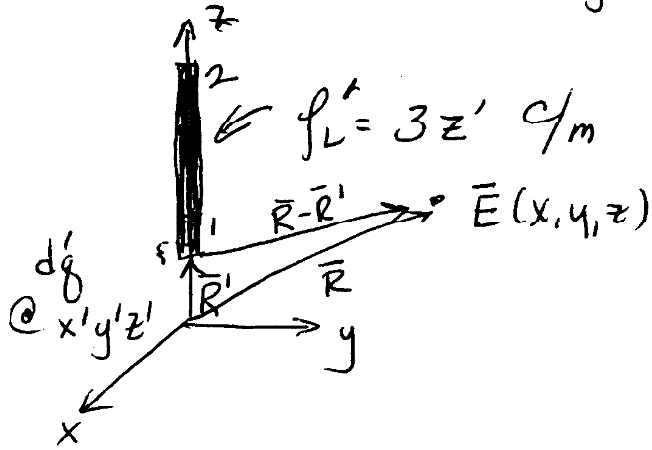
Example of Coulomb's Law
Linear Rectangular



① Vector from origin to charge (primed coordinates)

$$\bar{R}' = \underset{\substack{\uparrow \\ 0}}{x'}\,\hat{x} + \underset{\substack{\uparrow \\ 0}}{y'}\,\hat{y} + z'\,\hat{z} = z'\,\hat{z}$$

For our geometry

② Vector from origin to field (unprimed coordinates)

$$\bar{R} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$$

③ Apply Coulomb's Law

$$d\bar{E} = \frac{dq'}{4\pi\varepsilon_0} \frac{\text{"}\hat{R}\text{"}}{\text{"}R\text{"}^2}$$

"$\hat{R}$" = unit vector from charge to field

$$= \frac{\bar{R}-\bar{R}'}{|\bar{R}-\bar{R}'|} = \frac{x\,\hat{x} + y\,\hat{y} + (z-z')\,\hat{z}}{[x^2 + y^2 + (z-z')^2]^{1/2}}$$

"R" = distance from charge to field $= |\bar{R}-\bar{R}'|$

$$= [x^2 + y^2 + (z-z')^2]^{1/2}$$

$\bar{R}-\bar{R}'$ = vector from charge to field (not unit vector)

$$dq' = \rho_L'\, dL' = (3z')\, dz'$$

because of our geometry

Linear-Rect | continued

$$\bar{E} = \frac{1}{4\pi\varepsilon_0} \int_{z'=1m}^{2m} (3z') \frac{x\hat{x} + y\hat{y} + (z-z')\hat{z}}{[x^2+y^2+(z-z')^2]^{3/2}} \, dz'$$

$$\bar{E}(x,y,z) = \frac{1}{4\pi\varepsilon_0}\left\{ \int_{z'=1m}^{2m} \frac{3z' x \, dz'}{[x^2+y^2+(z-z')^2]^{3/2}} \hat{x} \right.$$

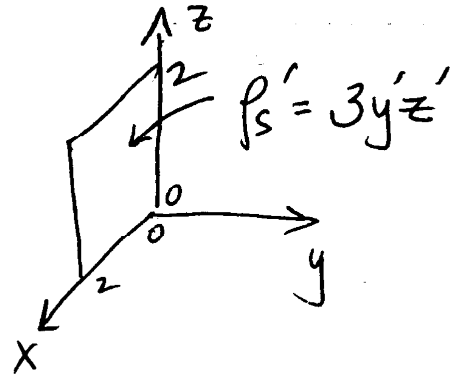
$$+ \int_{z'=1}^{2} \frac{3z' y \, dz'}{[x^2+y^2+(z-z')^2]^{3/2}} \hat{y}$$

$$\left. + \int_{z'=1}^{2} \frac{3z' (z-z')}{[x^2+y^2+(z-z')^2]^{3/2}} \hat{z} \right\}$$

If location of $\bar{E}$ is given, substitute this (xyz) for x, y, z

3 components of $\bar{E}$

Example of Coulomb's Law
Surface Rectangular



① Vector from origin to charge

$$\bar{R}' = x'\hat{x} + y'\hat{y} + z'\hat{z} = x'\hat{x} + z'\hat{z}$$

$y' = 0$ because of our geometry

② Vector from origin to field

$$\bar{R} = x\hat{x} + y\hat{y} + z\hat{z}$$

③ Coulomb's Law

$$d\bar{E} = \frac{dq' \; \text{“}\hat{R}\text{”}}{4\pi\varepsilon_0 \; \text{“}R\text{”}^2}$$

$$\text{“}\hat{R}\text{”} = \text{unit vector from charge to field}$$

$$= \frac{\bar{R} - \bar{R}'}{|\bar{R} - \bar{R}'|} = \frac{(x-x')\hat{x} + y\hat{y} + (z-z')\hat{z}}{[(x-x')^2 + y^2 + (z-z')^2]^{1/2}}$$

$$\text{“}R\text{”} = \text{distance from charge to field}$$

$$= |\bar{R} - \bar{R}'| = [(x-x')^2 + y^2 + (z-z')^2]^{1/2}$$

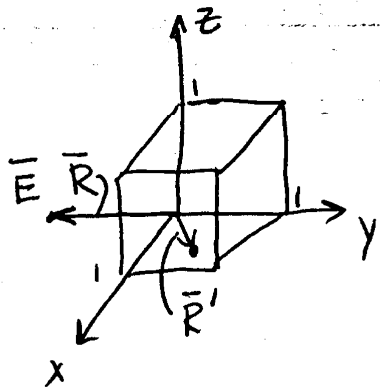
$$dq' = \rho_s' \, ds' = (3y'z') \underbrace{dx'dz'}_{}$$

because of our geometry

$$\bar{E} = \frac{1}{4\pi\varepsilon_0} \int_{x'=0}^{2m} \int_{z'=0}^{2m} (3y'z') \frac{(x-x')\hat{x} + y\hat{y} + (z-z')\hat{z}}{((x-x')^2 + y^2 + (z-z')^2)^{3/2}} \, dx'dz'$$

Examples of Coulomb's Law

Volume charge distribution in rectangular coord.



Given: $\rho_v = \frac{x'^2 y'}{z'} \frac{C}{m^3}$ (Non-uniform charge distribution)

① Vector from origin to charge point

$\bar{R}' = x'\hat{x} + y'\hat{y} + z'\hat{z}$

use "primed" coordinates & variables for charges

② Vector from origin to $\bar{E}$ field point

$\bar{R} = x\hat{x} + y\hat{y} + z\hat{z}$

use "unprimed" coordinates & variables for field pt.

③ Apply Coulomb's Law

$$d\bar{E} = \frac{dq'}{4\pi\varepsilon_0 \text{"}R\text{"}^2} \text{"}\hat{R}\text{"}$$

unit vector from charge to field point

distance from charge to field point

$$\text{"}\hat{R}\text{"} = \frac{\bar{R}-\bar{R}'}{|\bar{R}-\bar{R}'|} = \frac{(x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}}{[(x-x')^2 + (y-y')^2 + (z-z')^2]^{1/2}}$$

$$\text{"}R\text{"} = |\bar{R}-\bar{R}'| = [(x-x')^2 + (y-y')^2 + (z-z')^2]^{1/2}$$

$$dq' = \rho_v' \, dv' = \underbrace{\frac{(x')^2 y'}{z'}}_{\rho_v' \text{ given above}} \underbrace{dx'\,dy'\,dz'}_{dv' \text{ see page 110}}$$

Coulomb - Volume - ~~Spherical~~ - Rectangular - cont

$$\bar{E} = \frac{1}{4\pi\varepsilon_0} \int_{x'=0}^{1} \int_{y'=0}^{1} \int_{z'=0}^{1}$$

← Limits on charge distribution

$$\left[\frac{(x')^2 y'}{z'}\right] \frac{(x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}}{\left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{3/2}} \, dx'dy'dz'$$

Integrate over charges

xyz are "constant" wrt x'y'z'

This gives $\hat{x}$ component of field

$$\bar{E} = \frac{1}{4\pi\varepsilon_0} \Big\{ \int_{x'=0}^{1} \int_{y'=0}^{1} \int_{z'=0}^{1} \frac{(x')^2 y'}{z'} \frac{(x-x')\,\hat{x}\; dx'dy'dz'}{\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{3/2}}$$

$$+ \int\int\int \frac{(x')^2 y'}{z'} \frac{(y-y')\,\hat{y}\; dx'dy'dz'}{\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{3/2}} \quad \leftarrow \hat{y} \text{ component}$$

$$+ \int\int\int \frac{(x')^2 y'}{z'} \frac{(z-z')\,\hat{z}\; dx'dy'dz'}{\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{3/2}} \quad \leftarrow \hat{z}\text{-component}$$

$$= E_x\,\hat{x} + E_y\,\hat{y} + E_z\,\hat{z}$$

these will be functions of x, y, z
<u>OR</u> xyz could be specified, and then you could find $\bar{E}$ at that point.